## Science Activities

# Levers and mixtures: An integrated science and mathematics activity to solve problems 

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# Levers and mixtures: An integrated science and mathematics activity to solve problems 

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#### Abstract

In recent years, the integration of science and mathematics has become popular among educators because of its potential benefits for student learning. The purpose of this study is to introduce a two-day interdisciplinary lesson that brings science and mathematics concepts together, actively engaging students in working with percentages of the ingredients in mixtures with the concept of torque. Participation in this Grade 7-9 lesson provides opportunities for students to learn from both content areas as they progress through a variety of science process skills.


## KEYWORDS

Integration of science and mathematics; lever;
mathematics education; mixture problems; physics education

## Introduction

In recent years, the integration of science and mathematics has become popular among educators because of its potential benefits for student learning. One benefit of integration is to increase students' conceptual understanding in both subject areas by combining hands-on materials typical for use in one subject with hands-on materials typical for use in another subject area (Treacy and O'Donoghue 2014). Davison, Miller, and Metheny (1995) defined content-specific integration as "choosing an existing curriculum objective from mathematics and one from science and planning an activity that involves instruction in each of these objectives" (227). This sort of integration activity can support an unfamiliar concept in one content area by using a concept in the other content area that students already know (Phillips and Jeffery 2016).

This article describes a combination activity, Levers and Mixtures, which combines the torque in a lever into the mixture problems in mathematics. This twoday interdisciplinary lesson brings science and mathematics concepts together, actively engaging students in working with percentages of the ingredients in mixtures with the concept of torque. Students balance the torques of checkers in a lever and compute the percentage of a "mixture" of two types of checkers.

Students use science to discover the proportional relationship among the magnitudes of the force and its distance from the fulcrum in a lever and then apply that information mathematically to solve mixture problems. Participation in the activity provides opportunities for students to learn from both content areas as they progress through a variety of science process skills. Underscored in the Next Generation Science Standards (NGSS, 2013), the process skills include observing, predicting, collecting, and interpreting data, comparing results from two sources, and preparing tables. Activities such as the one described here have the potential to (a) increase students' problemsolving strategies in mathematics and science; and (b) increase their self-efficacy and achievements in highstakes examinations such as the SAT.

## Background and objectives

A lever was described by Renaissance scientists as one of the six classical simple machines. A lever generally consists of a rod pivoted at a fixed fulcrum. The ideal lever does not dissipate or accumulate the energy. The mechanical advantage of a lever can be determined by computing the balance of moments about the fulcrum. The torque of each force in the lever can be computed by multiplying the magnitude of the force with the

[^0]

Figure 1. Relations between the magnitudes of the applied forces and the distance of each force from the fulcrum point.
distance from the fulcrum to the point where the force is applied. With a balanced lever, the torque of each force in both sides should be equal. If $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ denote the magnitudes of the applied force at each side, and $\mathrm{d}_{1}$ and $\mathrm{d}_{2}$ represent the distance of each force from the fulcrum point, respectively, an inverse proportion exists when the lever is balanced (Figure 1).

$$
\begin{equation*}
\frac{F 1}{F 2}=\frac{d 2}{d 1} \tag{Equation1}
\end{equation*}
$$

Davison, Miller, and Metheny (1995) showed that the lever can be used to teach the concept of inverse proportion in mathematics. These researchers described a lever activity integrating torque concepts in science with proportion concepts in mathematics. In the activity, students used meter sticks, a fulcrum, and various metric weights to balance the lever and to determine the interplay between the weights of a body and the distances from the fulcrum.

Mixture problems are important in teaching the concept of ratio in mathematics education that continue throughout $\mathrm{K}-12$ education. At elementary grades, the concept of ratio begins with students computing the percentage of ingredients in solutions. In middle and high schools, students learn more about the concept by calculating the percentage of a resulting mixture when two or more solutions are mixed. Mixture problems in mathematics address essential mathematical process standards, underscored by the Common Core State Standards (CCSS, 2010). Mathematical process standards include (a) recognizing reasoning and proof as fundamental aspects of mathematics, (b) applying and adapting a variety of appropriate strategies to solve problems, (c) developing high-order thinking, (d) making and investigating mathematical conjectures, and (e) selecting, applying, and translating among mathematical representations to solve problems. In science, mixture problems also are important and are used to enhance students' understanding of the properties of matter.

Furthermore, students are more likely to confront mixture problems in standardized tests, which range from elementary schools through graduate schools.

Mixture problems can use ingredients of various types and different concentrations that are mixed together. The following mixture problem is a typical example for high school students:

> A pharmacist mixed some $10 \%$-saline solution with some $15 \%$-saline solution to obtain 100 mL of a $12 \%$ saline solution. How much of the $10 \%$-saline solution did the pharmacist use in the mixture? (California Department of Education, 2008, http://www.cde.ca.gov/ $\mathrm{ta} / \mathrm{tg} / \mathrm{sr} /$ documents/rtqalg1.pdf)

Steig (1999) described the typical way of solving mixture problems. First, compute the amount of the key item in each mixture. Then, sum them up to find the amount of the key item in the resulting mixture. Next, add the volume or mass of each mixture. Finally, divide the total amount of the key item by the total volume or mass.

In the computation below, A and B denote the volume or mass of Mixture 1 and Mixture 2, respectively; " $x$ " and " $y$ " denote the percentage of key item in Mixture 1 and Mixture 2, respectively, and " z " denotes the percentage of key item in the resulting mixture. In the computation below, we solve for z in the form of $\mathrm{A}, \mathrm{B}$, x , and y .

First, the amount of key item in each mixture is computed by:
$\mathrm{A} * \mathrm{x}=$ the amount of key item in Mixture 1,
and, $\mathrm{B} * \mathrm{y}=$ the amount of key item in Mixture 2

Then, let us sum them up:

$$
\begin{gathered}
(\mathrm{A} * \mathrm{x})+(\mathrm{B} * \mathrm{y})=\text { the amount of key item in the } \\
\text { resulting mixture }
\end{gathered}
$$

Now, let us compute the volume or mass of the resulting mixture by adding A and B :
$A+B=$ the volume or mass of the resulting mixture

Finally, z is computed by dividing the amount of key item in the resulting mixture by its volume or
mass:

$$
\begin{equation*}
z=\frac{((\mathrm{A} * \mathrm{x})+(\mathrm{B} * \mathrm{y}))}{A+B} \tag{Equation2}
\end{equation*}
$$

If we make an interior-exterior multiplication:

$$
(\mathrm{z} * \mathrm{~A})+(\mathrm{z} * \mathrm{~B})=(\mathrm{A} * \mathrm{x})+(\mathrm{B} * \mathrm{y})
$$

Then we group $A$ at one side of the equation and $B$ at the other side:

$$
\begin{equation*}
\mathrm{A} *(\mathrm{z}-\mathrm{x})=\mathrm{B} *(\mathrm{y}-\mathrm{z}) \tag{Equation3}
\end{equation*}
$$

Next, when A and B are put at one side, we find that

$$
\begin{equation*}
\frac{A}{B}=\frac{y-z}{z-x} \tag{Equation4}
\end{equation*}
$$

Equation 4 shows that the ratio between A and B inversely equals the ratio between $z-x$ and $y-z$. This implies that the ratio between the quantities of either volume or weights determines the percentage of the resulting mixture. Therefore, Equation 4 looks like Equation 1 because at a balanced lever, the ratio between $F_{1}$ and $F_{2}$ is equal to the inverse ratio between the distances of each force from the fulcrum. This lever example described by Davison and his colleagues (1995) can be used to solve mixture problems in mathematics. Along with the science and mathematics process skills explained above, one of the main objectives of this article is to describe a way to help students learn a problem-solving technique for mixture problems in mathematics.

Let us solve the mixture problem above by using the lever technique by considering " $z$ " as the equilibrium
point, " $A$ " as $F_{1}$, " $B$ " as $F_{2}$, and " $x-z$ " $d_{1}$ and " $y-z$ " as $\mathrm{d}_{2}$ (see Figure 2).

In the mixture question above, $\mathrm{A}+\mathrm{B}=100 \mathrm{ml}, \mathrm{x}=$ $10, \mathrm{z}=12$, and $\mathrm{y}=15$ are given. Let us put what we have at Equation 4:

$$
\begin{gathered}
\frac{A}{B}=\frac{y-z}{z-x} \\
\frac{A}{B}=\frac{15-12}{12-10} \\
\frac{A}{B}=\frac{3}{2}
\end{gathered}
$$

We know $\mathrm{A}+\mathrm{B}=100$, then
$A=60 \mathrm{ml}$ and $B=40 \mathrm{ml}$ are found.
As seen, such a problem-solving technique for mixture problems would be easier than the usual way of solving mixture problems in mathematics detailed by Steig (1999). This hands-on activity provides an opportunity for students to directly observe that the equilibrium point is closer to the bigger force and the percentage of the resulting solution is closer to the solution that is heavier when two solutions are added together.

As teachers, one of our most important purposes is to increase students' content knowledge and thus assure their success on tests. We also want students to increase their problem-solving skills in mathematics, which have direct effects on their problem solving in physics, chemistry, and biology. This activity has the potential to increase students' problem-solving skills and achievement in mathematics and science. Furthermore, this activity also provides opportunities for


Figure 2. Representation of a mixture problem as a torque in a lever.
students to compare and discuss the results from mathematical calculations and scientific experiments.

## Activity

## Materials

Students may bring materials from home, if necessary. The following materials are required for each group of the students:

1. wooden meter stick with 100 cm long
2. fulcrum (a hard-covered book)
3. total of 30 checkers ( 15 white and 15 black ones)
4. a handout (see Appendix)

I used a $100-\mathrm{cm}$ long wooden meter stick, which makes it easy to point out percentages represented on the hundred-scale. If a shorter meter stick is used, note that the distance should be converted to 100 cm . For the fulcrum, I used a hard-covered book. I used checkers for the representations of mixtures because (a) they are safe to have in the activity, (b) all checkers have identical weights, (c) they allow visualization of the mixture, and (d) they can be easily built up on the top of each other. Select two different colors for the checkers. Other materials that have a good geometric shape and identical weights such as Lego, dimes, or quarters can be used as well (see Figure 3). If teachers have a limited number of checkers, fewer than 30 checkers may be used. To track students' conceptual understanding, I prepared a handout (see Appendix) that guides students to have a discussion about what they found out after each of five phases of the activity. The handout provides instructions for the discussion and a place for them to write their answers.

## Time and grade level

This activity requires a minimum of two 40 -min class periods. During the activity, students can work in collaborative groups of three or four. The activity is suitable for students in Grades 7-9, as students are expected to learn the idea of the ratio and the proportional relationship in mathematics starting from Grade 6 and of torque in science starting from Grade 7.

## Stages of the activity

Five stages exist in this activity that go from more simple and concrete to more complex and abstract (see Table 1). The activity begins with a stage for initial exploration of the torque in a lever in which students


Figure 3. Materials used in the activity.
investigate the inverse ratio between the force and the distance from the fulcrum (Stage 1). Then students explore when two mixtures are mixed up; the ratio between A and B inversely equals the ratio between z $x$ and $y-z$. The next stage allows students to combine the first stage with the second one. Then students explore how the weight of the meter stick may influence lever experiments. Finally, the fifth stage is an evaluation phase that allows students to evaluate what they have learned from this activity.

## First stage (15 min)

Organizing questions. What is the relationship between the numbers of the checkers and the distance from the fulcrum? Is the relationship an inverse or direct proportional? Which the group of checkers is the fulcrum closer to? Why?

Procedures. The activity begins with an exploration of the relationship between the weight of a particle and its distance from the fulcrum in a lever. After introducing the materials, teachers ask students to place the fulcrum at the middle of the lever, to keep the lever stable, and to observe its movements. The teacher provides some time for students to investigate the behavior of the meter stick on the book. Then students put some checkers on the meter stick on either side of the fulcrum, first predicting and then finding the required checkers and its distance at the other side to keep the meter stick stable. Teachers remind students that the stability means that any side of the meter stick is not moving and the meter stick stays parallel to the ground. Students repeat this step several times with different amounts of checkers to explore the

Table 1. Summary of stages of the integrated activity.

| Stage | Time (min) | Purpose |
| :--- | :---: | :--- |
| 1 | 15 | To build a "balanced" lever using different <br> numbers of checkers. |
| 2 | 15 | To explore the ratio between $A$ and $B$ equal to <br> the ratio between $x-z$ and $y-z$. |
| To find $z$ in the previous stage by constructing a |  |  |
| balanced lever. |  |  |$\quad$| Stating the equation that the ratio between the numbers of checkers inversely |
| :---: |
| equals the ratio between their distances from the fulcrum. |
| Expressing an understanding that the ratio between the numbers of checkers |
| inversely equals the ratio between their distances from the fulcrum. |
| Expressing an understanding that the point where the fulcrum is located shows |
| the " $z$ " value in the resulting mixture. |

relationships between the number of checkers and their distance to the fulcrum. In small groups, students discuss the ratio between number of checkers and the distances to the fulcrum and then reach an agreement about their answers to the questions in the First Stage activity sheet questions (see Figure 4).

## Second stage (15 min)

Organizing questions. What is the relationship between the number of checkers in Mixture A and B and the percentages when Mixture A and B are added together? Which mixture (Mixture A or Mixture B) is the percentage the resulting mixture closer to? Why?

Procedures. The second stage helps students explore how the percentage of the ingredients in the resulting mixture determines the ratio among the amount of Mixture 1 and Mixture 2. Students create two groups of mixtures using all checkers (the number of white checkers should equal to the number of black checkers). Teachers help students imagine that they have saline solution where white checkers stands for salt and black ones for water. Then students calculate the percentage of the white checkers in each mixture as in
saline solution. Students have some time to think about what $\mathrm{x}, \mathrm{y}$, and z are. This stage helps students understand the percentage changes and the role of ratio in the resulting mixture before and after mixtures have been added up ( $|\mathrm{z}-\mathrm{x}|$, and $|\mathrm{y}-\mathrm{z}|$ distances in Figure 2).

## Third stage (20 min)

Organizing questions. What is the relationship between the number of checkers in the mixtures and the distance from the fulcrum? Is it an inverse or direct proportional? Which the mixture is the fulcrum closer to? Why?

Procedure. The third stage combines the first stage with the second one. Students re-create the mixtures they use in Stage 2 and reinstall the experiment in Stage 1. Because they use the same amount of white and black checkers in each mixture, they already know about the percentage of the resulting mixture. Thus students already know that the ratio determines the percentage in the resulting mixture. This stage helps students explore that the ratio between amounts of Mixture 1 and 2 is like the ratio between the


Figure 4. Students balancing the meter stick with checkers in Stage 1.


Figure 5. Students balancing the meter stick with checkers in Stage 3.
numbers of checkers in a balanced lever. To do this, students put each mixture on the centimeter reading that corresponds to the white checker percentage value of the mixtures on the meter stick. Let us say that students have prepared a mixture with two white and eight black checkers. The percentage of white checkers on this mixture is $20 \%$. So students put this mixture of checkers on 20 cm on the meter stick. The other mixture also is placed on the centimeter value that corresponded to its percentage on the meter stick. Then the students find the equilibrium point by moving the fulcrum back and forth. Students note the centimeter value of where the fulcrum placed on the meter stick. Students calculate the ratio between the number of checkers at the left side and the right side $\left(\mathrm{N}_{1} / \mathrm{N}_{2}\right)$, and the ratio between their distances to the fulcrum (see Table 3 in Appendix). Students have some time to share what they have found with the other mates.

The students should find the $50-\mathrm{cm}$ point on the meter stick as the equilibrium point because they used all checkers (Figure 5). Before completing this stage, the teacher asks a critical question: "Why did all find the equilibrium point as 50 cm even though they had
prepared different mixtures?" Students compare their results (the ratios in tables) with Stage 2. At the beginning, the students may not be able to answer this question. They may need to elaborate on what they have done in the first and second stages. Then the teacher can encourage them to think what $\mathrm{N}_{1}$ and $|\mathrm{AO}|$, and $\mathrm{N}_{2}$ and $|\mathrm{OB}|$ are, compared with $\mathrm{x}, \mathrm{y}$, and z in Stage 2. After a while, most students in my classes said that the equilibrium point displays the white checker percentage in the resulting mixture. As they had used all of the checkers, they had the same amount of white and black checkers, and the midpoint was, therefore, 50 cm .

## Fourth stage (15 min)

Organizing questions. Why is the equilibrium point different from 50 cm ? Why is the mathematical result different from the value on the meter stick? What do these differences tell us?

Procedure. The purpose of this stage is to show students that the weight of the meter stick may influence lever experiments. As we had equal number of white and black checkers during the previous stages, the


Figure 6. Students balancing the meter stick with checkers in Stage 4.


Figure 7. Students balancing the meter stick with checkers in Stage 5.
equilibrium point was 50 cm (i.e., where the center of gravity is). This stage will show students that if they do not use equal number of white and black checkers, the equilibrium point will be different from 50 cm , and the weight of the meter stick will influence the balance in the meter stick. By doing so, students will learn limits of their experiment. First, students remove three black checkers from their box (not used in the stage). Then they prepare two mixtures with all checkers (except three black checkers) and calculate the percentages of white checkers in each. Then, as in the second stage, they mix them up and calculate the white checker percentage of the resulting mixture and note it as the mathematical result in Table 4. Next, they recreate the same mixtures and do what they did in Stage 3 to find the white checker percentage of the resulting mixture and note it as the experimental result in the Table 4. If the equilibrium point is not at 50 cm , the students realized that the equilibrium point and mathematical calculation are different due to the weight of the meter stick (see Figure 6). Students discuss why the answer was different and the technique does not work in this time. Students make a group discussion and come up with an answer. After the students notice that the weight of the meter stick affects
the balance, the students are asked how many white and black checkers correspond to the meter stick.

## Fifth stage (15 min)

Organizing questions. What is the connection between science and mathematics in the activity? How can we use mathematics for science or vice versa?

Procedure. At the last stage of the activity, the teacher can assign an evaluation to determine whether students have learned the relationship between the numbers of checkers and the equilibrium point. In this stage, students are given Mixture A and the resulting mixture and asked to find two different Mixture Bs. By doing so, this stage helps students to explore that there would be multiple solutions in science and mathematics. For this purpose, place ten checkers (three black and seven white checkers) on 70 cm , and the fulcrum on 50 cm . Then students find two different mixtures to make the white checker percentage of the resulting mixture $50 \%$. Students' drawings on the handout showed that they consider this technique to solve mixture problems (Figure 7 and 8).


Figure 8. A student solving a mixture problem with torque technique.

## Conclusion

This activity integrates the torque in a lever with solving mixture problems in mathematics. As a hands-on activity, it promotes situated learning and critical thinking. The goal of the lesson is for students to understand that there is an inverse proportion between the distance and weights on a lever and that the concept of inverse proportion can be used to solve mixture problems in mathematics. This activity has potential to provide students a variety of science process skills underscored in the NGSS, including observing, predicting, collecting, and interpreting data, comparing results from two sources, and preparing tables, and mathematical skills underscored by the CCSS, including apply and adapt a variety of appropriate strategies to solve problems, high-order thinking and selecting, applying, and translating among mathematical representations to solve problems. This activity allows teachers to support students' mathematical knowledge with their scientific knowledge and their scientific ideas with mathematical representation. Furthermore, this activity may help students have a notion that science and mathematics are connected to each other and do not exist in the world as separated from one another but combined. Combined, mathematics and science enable students to progressively develop more conceptually abstract ideas by exploring relationships between them by using hands-on materials.

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## Appendix

## Stage 1

- At the first step, let us put the book in the center of the meter stick as the picture and try to keep the meter stick stable.
- Now, while one friend keeps one side of the meter stick stable, place some checkers on the meter stick at the left side of the book.
- Then predict how many checkers and where should be put at the right side of the book to keep the meter stick stable.
- Try to balance the meter stick by adding or removing any checker at any side of the book or by moving the checkers. (At balance, any side of the meter stick is not moving and the meter stick stays parallel to the ground.)
- Record the number of the checkers at any side and its distance to the fulcrum $(|A O|$ and $|O B|$ in the picture).
- Repeat this step several times and fill in Table 1.


Table 1.
Fulcrum
50 cm
50 cm
50 cm
50 cm

$\mathrm{~N}_{\mathrm{L}}=$ number of checkers at the left, $|\mathrm{AO}|=$ the distance of the checkers at
left to the fulcrum.
$\mathrm{N}_{\mathrm{R}}=$ number of checkers at the right, $|\mathrm{OB}|=$ the distance of the checkers at
right to the fulcrum.

## Now, let us answer the following questions.

1. What is the relationship between the numbers of the checkers and the distance from the fulcrum?
$\qquad$
2. Is the relationship an inverse or direct proportional? Why?
3. Which group of checkers is the fulcrum close to? Could you explain your answer?
4. How can you show the relationship among $|\mathrm{AO}|, \mathrm{N}_{\mathrm{L}}$, $|\mathrm{OB}|$, and $\mathrm{N}_{\mathrm{R}}$ ?

## Stage 2

- Let's split all checkers into two groups (each group contains white and black checkers) and name them Mixture 1 and Mixture 2.
- Then, record the number of the checkers (A and B) and the white checkers in each mixture in Table 2.
- Next, calculate the percentage of the white checkers in each mixture ( x and y ) and note it in Table 2.
- Now, mix two these mixtures of checkers together and name it as the resulting mixture.
- Then, record the number and percentage ( z ) of white checkers in the resulting mixture.
- Next, let's compute the amount of the change on the percentage of white checkers in Mixture 1 when we mix it up and record it as $|\mathrm{z}-\mathrm{x}|$ in Table 2.
- Next, let's compute the amount of the change on the percentage of white checkers in Mixture 2 when we mix it up and record it as $|z-y|$ in Table 2.
- Then, compute the ratio between A and B and $|z-y|$ and $|z-\mathrm{x}|$.
- Let's repeat this step several times and fill in Table 2.

Table 2.


Let's answer the following questions.

1. To which of x and y is z closer? Why?
2. What is the relationship between $\frac{N_{1}}{N_{2}}$ and $\frac{|y-z|}{|z-x|}$ ? Why?

## Stage 3

- At the first step, let us set up the book and the meter stick as in Activity 1 and try to keep the meter stick stable.
- Then, let us split the checkers into the same groups in Activity 2.
- We already know the percentage of white checkers in each mixture from Table 2 ( x and y ).
- Now let us place Mixture 1 on the centimeter reading that corresponds to the " $x$ " value on the meter stick.
- Let's say if Mixture 1 has two white and eight black checkers, the white percentage is $20 \%$ and place it on 20 cm on the meter stick.
- Then, do it for Mixture 2.
- Now, balance the meter stick by moving it on the fulcrum.
- Next, record the point where the fulcrum stands at the balance, $\overline{\mathrm{AO}}, \overline{\mathrm{OB}}, \mathrm{N}_{1}$, and $\mathrm{N}_{2}$.
- Then, compute the ratio between $\mathrm{N}_{1}$ and $\mathrm{N}_{2}$ and the ratio between $\overline{\mathrm{AO}}$ and $\overline{\mathrm{OB}}$.
- Let's repeat this step several times with numbers in Table 2.

Table 3.

| Trials | Fulcrum (cm) | Mixture A |  |  | ure B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{N}_{1}$ | \| $\mathrm{AO} \mid$ | $\mathrm{N}_{2}$ | \|OB| | $\mathrm{N}_{1} / \mathrm{N}_{2}$ | $\|A O\| /\|O B\|$ |
| 1. Trial |  |  |  |  |  |  |  |
| 2. Trial |  |  |  |  |  |  |  |
| 3. Trial |  |  |  |  |  |  |  |
| 4. Trial |  |  |  |  |  |  |  |

Let's answer the following questions:

1. What is the relationship between the number of checkers in the mixtures and the distance from the fulcrum?
2. Which mixture is the fulcrum closer to? Why?
3. Is the relationship inverse or direct proportional? Why?
4. Compare the ratio values you found in Activity 2 and Activity 3. Are they the same? Remember what z, y, and x are in Activity 2 and what they stand for in Activity 3.
5. How may this help you for solving mixture problem in mathematics by considering what we have done so far? Discuss with your friends.

## Stage 4

- At the first step, let's remove three black checkers from our inventory and not use them during this activity.
- Then, let us split the checkers into two groups you call.
- Next, calculate the percentage of the white checkers in each mixture as in Activity 2.
- Then, mix the group of checkers together and calculate the percentage of the white checkers in the resulting mixture and note it in the mathematical result column in Table 4.
- Next, separate the mixture back into two groups (the same group at the beginning) and place them in the meter stick as Activity 3.
- Let's balance the meter stick by moving the fulcrum back and forth.
- Now, record the point where the fulcrum stands at the balance in the experimental result column.
- Let's repeat this step several times and fill in Table 4.

Table 4.


## Let's answer the following questions.

1. Is any experimental result different from the mathematical one?
2. If there is, why do you think they are different?
3. What you do think is different from Activity 2 and 3?

Table 5.

| The fulcrum | Mixture A |  |  | Mixture B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | \# of whites | \# of blacks | Cm value | \# of whites | \# of blacks | Cm value |
| 50 cm | 7 | 3 | 70 |  |  |  |
| 50 cm | 7 | 3 | 70 |  |  |  |
| 50 cm | 7 | 3 | 70 |  |  |  |
| 50 cm | 6 | 4 | 60 |  |  |  |
| 50 cm | 6 | 4 | 60 |  |  |  |
| 50 cm | 6 | 4 | 60 |  |  |  |

4. Let's imagine the meter stick as a mixture consisting of white and black checkers. Could you find how many white and black checkers correspond to the meter stick? (Hint: Use the relationship between the number of checkers and the distance to the fulcrum in Activity 1)

## Stage 5

- In this activity, let us set up the book and meter stick as Activity 1.
- Then, pick seven white and three black checkers and place them on 70 cm on the meter stick.
- Now, let us find three different mixtures that balance the meter stick.
- Repeat this step for six white and four black checkers.
(Hint: First identify which ones of $|\mathrm{AO}|, \mathrm{A},|\mathrm{OB}|$, and B are known.)

Let's try to solve the following mathematics questions:
xkg is taken from the flour-sugar mixture, which is $70 \%$ sugar by weight, and y kg is taken from another flour-sugar mixture, which is $45 \%$ sugar, and they are mixed up. The resulting mixture with $65 \%$ sugar by weight is obtained. How many times is $x, y$ ?
a) 2
b) 3
c) 4
d) 5
e) 7


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