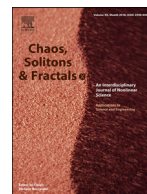




Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Forecasting the cumulative number of confirmed cases of COVID-19 in Italy, UK and USA using fractional nonlinear grey Bernoulli model

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ARTICLE INFO

Article history:

Received 19 April 2020

Revised 23 April 2020

Accepted 26 May 2020

Available online 29 May 2020

Keywords:

COVID-19

Italy

UK

USA

Forecasting

Fractional grey model

ABSTRACT

Since the new coronavirus (COVID-19) outbreak spread from China to other countries, it has been a curiosity for how and how long the number of cases will increase. This study aims to forecast the number of confirmed cases of COVID-19 in Italy, the United Kingdom (UK) and the United States of America (USA). In this study, grey model (GM(1,1)), nonlinear grey Bernoulli model (NGBM(1,1)) and fractional nonlinear grey Bernoulli model (FANGBM(1,1)) are compared for the prediction. Therefore, grey prediction models, especially the fractional accumulated grey model, are used for the first time in this topic and it is believed that this study fills the gap in the literature. This model is applied to predict the data for the period 19/03–22/04/2020 (35 days) and forecast the data for the period 23/04–22/05/2020. The number of cases of COVID-19 in these countries are handled cumulatively. The prediction performance of the models is measured by the calculation of root mean square error (RMSE), mean absolute percentage error (MAPE) and R^2 values. It is obtained that FANGBM(1,1) gives the highest prediction performance with having the lowest RMSE and MAPE values and the highest R^2 values for these countries. Results show that the cumulative number of cases for Italy, UK and USA is forecasted to be about 233000, 189000 and 1160000, respectively, on May 22, 2020 which corresponds to the average daily rate is 0.80%, 1.19% and 1.13%, respectively, from 22/04/2020 to 22/05/2020. The FANGBM(1,1) presents that the cumulative number of cases of COVID-19 increases at a diminishing rate from 23/04/2020 to 22/05/2020 for these countries.

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1. Introduction

The new coronavirus (COVID-19) outbreak first appeared allegedly in Wuhan, China in December 2019. There have been reported about 2.1 million confirmed COVID-19 cases and 146 thousand deaths worldwide since then. China, Europe and USA have been the centre of this outbreak, respectively. Italy and the United Kingdom are most affected countries in Europe by this outbreak [21]. Italy has the largest number of the cumulative confirmed cases of COVID-19 with 86500 cases as of 28/03/2020 in the world. Furthermore, USA ranks the first place with the most cumulative confirmed cases as of 22/04/2020 [22]. Therefore, it may be vital research topic to estimate how far the outbreak would spread in a specific country or worldwide.

Recently, many researchers have focused on forecasting the number of COVID-19 cases accurately. Fanelli and Piazza [7] used

susceptible-infected-recovered-deaths (SIRD) model to forecast of COVID-19 spreading for Italy, China and France. Al-qaness et al. [1] used an improved adaptive neuro-fuzzy inference system (ANFIS), called FPASSA-ANFIS, to forecast confirmed cases of COVID-19 in China. Roosa et al. [17] employed a generalized logistic growth model, the Richards growth model, and a sub-epidemic wave model to forecast the cumulative cases in the provinces of Guangdong and Zhejiang, China. Petropoulos and Makridakis [16] used logistics curve (S-Curve), which is a kind of time series approaches, to forecast the cumulative confirmed cases globally. Benvenuto et al. [3] used Auto Regressive Integrated Moving Average (ARIMA) model to forecast the prevalence and incidence of COVID-19. Magal and Webb [13] used a model which consists of ordinary differential equations, to forecast the cumulative reported and unreported cases for COVID-19 in South Korea, Italy, France and Germany. Zhang et al. [27] used a segmented Poisson model to forecast the daily new cases data of the COVID-19 in Canada, France, Germany, Italy, UK and USA. They tried to determine turning point, duration and attack rate of COVID-19 in these countries. To sum up, the studies on forecasting the number of cases

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Nomenclatures	
AGO	Accumulated generating operation
APE	Absolute percentage error (%)
GM(1,1)	Basic grey model
FANGBM(1,1)	Fractional nonlinear grey Bernoulli model
MAPE	Mean absolute percentage error (%)
NGBM(1,1)	Nonlinear grey Bernoulli model
R ²	Squared correlation coefficient
RMSE	Root mean square error
X ⁽⁰⁾	The original data sequence
n	Length of data
γ	Power index value
r	Fractional order value

of COVID-19 pandemic have been interesting, important and trending. However, it can be argued that only a few forecasting methods have been employed for the prediction of COVID-19 cases in the literature. Therefore, the methods used to estimate prospective cases in the future need to be diversified. To fill the gap in the literature, the fractional accumulated grey prediction models are employed for forecasting COVID-19 cases in Italy, UK and USA in this study.

The fractional accumulated grey prediction models have been widely used as a forecasting tool due to its high prediction performance by many researchers recently ([8,14,24] and [23]). One of these is the fractional nonlinear grey Bernoulli model (FANGBM(1,1)), which is proposed by Wu et al. [25]. The FANGBM(1,1) is a highly improved form of the basic grey prediction model (GM(1,1)) which is firstly proposed by Julong Deng in 1982 [5]. The FANGBM(1,1) is the combination of the r-th accumulated generation operation and nonlinear grey Bernoulli model (NGBM(1,1)). Two parameters, which are fractional order value (r) and power index value (γ), determine the prediction characteristic of this model. Also, it is known that FANGBM(1,1) reduces to NGBM(1,1) when γ ≠ 0 and r = 1 and reduces to the GM(1,1) when γ = 0 and r = 1 [25]. Wu et al. [25] used the FANGBM(1,1) to forecast China's total renewable energy consumption, hydroelectricity consumption, wind consumption, solar consumption, and consumption of other renewable energies. Şahin [18] used the FANGBM(1,1) to forecast Turkey's electricity generation and installed capacity from total renewable and hydro energy. The common feature of these two studies is that the FANGBM(1,1) gives the best prediction result among other models. Additionally, this model has not yet been used as a forecasting method in the field of the infectious diseases.

To benefit from the powerful forecasting nature of the fractional grey prediction model is the main motivation of this study for predicting the number of cases of COVID-19. This study aims at forecasting the number of cases of COVID-19 in Italy, UK and USA using FANGBM(1,1), which is an improved grey prediction model. Two parameters of the FANGBM(1,1), which are r and γ, are optimized using genetic algorithm (GA) technique for this study.

$$A^{-r} = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -r & 1 & 0 & \dots & 0 \\ \frac{r(r-1)}{2!} & -r & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{(r(r-1)\dots(r-n+2))(-1)^{n-1}}{(n-1)!} & \frac{(r(r-1)\dots(r-n+3))(-1)^{n-2}}{(n-2)!} & \frac{(r(r-1)\dots(r-n+4))(-1)^{n-2}}{(n-3)!} & \dots & 1 \end{pmatrix} \tag{5}$$

The main contributions of this study are:

- As far as is known, the FANGBM(1,1) model is used for the first time in forecasting the number of cases COVID-19. Therefore, it is believed that this study fills the gap in the literature.
- The results of this study are expected to help the governments in developing their own policies to take appropriate measures regarding COVID-19 outbreaks at this time when more information is needed on time.
- Additionally, it is believed that this study will be one of the alternative and guiding studies for estimating the number of cases of COVID-19 for other countries or provinces.

The remainder of this paper is structured as follows. Section 2 gives the methodology of grey prediction models. Section 3 specifies how optimal parameters are obtained and prediction performance is measured. Section 4 presents the results and discusses the results of this study. In section 5, the main conclusions of this study and suggestions for further studies are mentioned.

2. The methodology of grey prediction models for this study

In this section, the mathematical form of fractional nonlinear grey Bernoulli model (FANGBM(1,1)), nonlinear grey Bernoulli model (NGBM(1,1)) and grey model (GM(1,1)) is presented.

2.1. The fractional nonlinear grey Bernoulli model (FANGBM(1,1))

The methodology of FANGBM(1,1) can be explained in the following steps [25].

Step 1: The original data sequence X⁽⁰⁾ is formed.

$$X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), X^{(0)}(3), \dots, X^{(0)}(n)\} \tag{1}$$

where n indicates the length of the sequence or the number of the original data.

Step 2: Transforming the X⁽⁰⁾ to the X^(r) using r-th accumulated generating operation (r-AGO) where r indicates the fractional order value r > 0.

$$X^{(r)}(k) = \sum_{i=1}^k X^{(r-1)}(i), \quad k = 1, 2, 3, \dots, n \tag{2}$$

And, X^(r) can be formed with matrix.

$$X^{(r)} = A^r X^{(0)} \tag{3}$$

where A^r and A^{-r} indicate the r-th order accumulated generating matrix and the inverse accumulated generating operation matrix, respectively. The form of A^r and A^{-r} can be given by the following equations [8].

$$A^r = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ r & 1 & 0 & \dots & 0 \\ \frac{r(r+1)}{2!} & r & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{r(r+1)\dots(r+n-2)}{(n-1)!} & \frac{r(r+1)\dots(r+n-3)}{(n-2)!} & \frac{r(r+1)\dots(r+n-3)}{(n-3)!} & \dots & 1 \end{pmatrix} \tag{4}$$

When $r = 1$, $X^{(r)}(k)$ turns into the first-order accumulated generating operation (1-AGO) sequence of $X^{(0)}$, called as $X^{(1)}(k)$. $X^{(1)}(k)$ can be formulated as,

$$X^{(1)}(k) = \sum_{i=1}^k X^{(0)}(i), \quad k = 1, 2, 3, \dots, n \tag{6}$$

The relationship between $X^{(0)}$ and $X^{(1)}$ can be given as,

$$X^{(0)} = A^{-1}X^{(1)} \tag{7}$$

where, A^{-1} is the inverse of the A matrix. The matrix form of A and A^{-1} can be given by the following equations:

$$A = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ 1 & 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 1 & 1 & 1 & \dots & 1 \end{pmatrix} \tag{8}$$

and,

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \tag{9}$$

Additionally, $X^{(0)}$ can be derived by various forms of the A matrix [8] as,

$$X^{(0)} = A^{-1}X^{(1)} = A^{-2}X^{(2)} = A^{-3}X^{(3)} = \dots = A^{-r}X^{(r)} \tag{10}$$

Step 3: Defining the whitening equation and the grey differential equation by the following equations.

$$\frac{dX^{(r)}(k)}{dt} + aX^{(r)}(k) = b(X^{(r)}(k))^\gamma, \quad r > 0 \tag{11}$$

$$X^{(r)}(k) - X^{(r)}(k-1) + az^{(r)}(k) = b(z^{(r)}(k))^\gamma \tag{12}$$

where γ indicates the power index value. In Eq. (12), $z^{(r)}(k)$ can be given as [14]:

$$z^{(r)}(k) = 0.5 * (X^{(r)}(k) + X^{(r)}(k-1)), \quad k = 2, 3, 4 \dots n \tag{13}$$

Step 4: Obtaining of parameters a and b using the least squares method:

$$\begin{bmatrix} a & b \end{bmatrix}^T = [B^T B]^{-1} B^T Y \tag{14}$$

where, B and Y are given by the following equations:

$$B = \begin{bmatrix} -z^{(r)}(2) & (z^{(r)}(2))^\gamma \\ -z^{(r)}(3) & (z^{(r)}(3))^\gamma \\ -z^{(r)}(4) & (z^{(r)}(4))^\gamma \\ \vdots & \vdots \\ \vdots & \vdots \\ -z^{(r)}(n) & (z^{(r)}(n))^\gamma \end{bmatrix} \tag{15}$$

and,

$$Y = \begin{bmatrix} X^{(r)}(2) - X^{(r)}(1) \\ X^{(r)}(3) - X^{(r)}(2) \\ X^{(r)}(4) - X^{(r)}(3) \\ \vdots \\ \vdots \\ X^{(r)}(n) - X^{(r)}(n-1) \end{bmatrix} \tag{16}$$

Step 5: Finally, calculation of the predicted values by the following equations:

$$\begin{cases} \hat{X}^{(r)}(1) = X^{(0)}(1) \\ \hat{X}^{(r)}(k) = \left[\left((\hat{X}^{(r)}(1))^{1-\gamma} - \frac{b}{a} \right) e^{-a*(1-\gamma)(k-1)} + \frac{b}{a} \right]^{\frac{1}{1-\gamma}}, \quad k = 2, 3, \dots, n \end{cases} \tag{17}$$

2.2. The nonlinear grey Bernoulli model (NGBM(1,1))

The structure of the nonlinear grey Bernoulli model NGBM(1,1) is defined as [4, 25]:

Step 1 and Step 2 is the same as the FANGBM(1,1).

Step 3: In NGBM(1,1), power index value and fractional order value are obtained as $\gamma \neq 0$ and $r = 1$, respectively. Therefore, the first-order differential equation or whitening equation is established as:

$$\frac{dX^{(1)}(k)}{dt} + aX^{(1)}(k) = b(X^{(1)}(k))^\gamma \tag{18}$$

Step 4 and 5 is the same as the FANGBM(1,1).

2.3. The basic grey model (GM(1,1))

The methodology of the GM(1,1) can be summarised as:

Step 1 and 2 is the same as the FANGBM(1,1).

Step 3: When $\gamma = 0$ and $r = 1$, the FANGBM(1,1) reduces to GM(1,1) which is firstly proposed by Deng [5]. The mathematical form of the first-order differential equation or whitening equation of GM(1,1) is:

$$\frac{dX^{(1)}(k)}{dt} + aX^{(1)}(k) = b \tag{19}$$

Step 4 and 5 is the same as the FANGBM(1,1).

3. Obtaining of the optimal parameters and statistical analysis

In this study, parameters γ and r of the FANGBM(1,1) and parameter γ of NGBM(1,1) are obtained by using genetic algorithm (GA) technique that is solved by a software package add-in Microsoft Excel 2016 [15]. To obtain the optimal parameters γ and r , the accuracy between the predicted value and the original value is measured by the calculation of absolute percentage error (APE). Additionally, mean absolute percentage error (MAPE) and root mean square error (RMSE) values are calculated to measure the prediction performance of this model. If MAPE is lower than 10%, the prediction model is classified as high level [11,20].

The calculation of APE, MAPE, RMSE and R^2 can be expressed by the following equations [1,6,19,26],

$$APE (\%) = \left| \frac{u(i) - \hat{u}(i)}{u(i)} \right| \times 100 \tag{20}$$

$$MAPE (\%) = \sum_{i=2}^n \left| \frac{u(i) - \hat{u}(i)}{u(i)} \right| \times \frac{100}{n-1} \tag{21}$$

$$RMSE = \sqrt{\frac{\sum_{i=2}^n (u(i) - \hat{u}(i))^2}{n-1}} \tag{22}$$

$$R^2 = 1 - \frac{\sum_{i=2}^n (u(i) - \hat{u}(i))^2}{\sum_{i=2}^n (u(i) - \bar{u})^2} \tag{23}$$

where, u , \hat{u} and \bar{u} denotes the original data, the predicted data and the average of the original data, respectively. It is known that the highest R^2 and the lowest MAPE and RMSE values define the model as the best prediction model [1].

Table 1
Results of the optimal parameters of NGBM(1,1) and FANGBM(1,1) for this study.

Parameters	NGBM(1,1)			FANGBM(1,1)		
	Italy	UK	USA	Italy	UK	USA
γ	0.3535	0.8024	0.8004	0.3159	1.1747	0.9231
r	1	1	1	0.1061	0.1716	0.2753

Table 3
MAPE, RMSE and R² results of the prediction models for the selected countries.

Countries	Prediction models	MAPE(%)	RMSE	R ²
Italy	GM(1,1)	9.2422	9846	0.9469
	NGBM(1,1)	1.9690	2418	0.9970
	FANGBM(1,1)	0.9174	1223	0.9993
UK	GM(1,1)	66.8486	18224	0.9686
	NGBM(1,1)	5.3279	1262	0.9993
	FANGBM(1,1)	2.8133	796	0.9996
USA	GM(1,1)	99.5818	118786	0.9581
	NGBM(1,1)	6.0082	6002	0.9995
	FANGBM(1,1)	4.8950	5767	0.9996

4. Results and Discussions

In this study, data of the cumulative confirmed cases of coronavirus (COVID-19) for Italy, the United Kingdom (UK) and the United States of America (USA) was taken from the database of World Health Organization [22]. Table 1 gives the optimal parameters of the FANGBM(1,1) for the prediction of the cumulative confirmed cases of COVID-19 in Italy, UK and USA for the period 20/03-22/04/2020. The optimal parameters of NGBM(1,1) and FANGBM(1,1) are obtained according to having minimum mean absolute percentage error (MAPE) value which is given in Eq. (21).

Table 2 presents the absolute percentage error (APE) values of GM(1,1), NGBM(1,1) and FANGBM(1,1) for Italy, UK and USA. It is known that the first APE value of GM(1,1), NGBM(1,1) and FANGBM(1,1) for the data 19/03/2020 is ignored for this study.

Results of the RMSE, MAPE and R² values of GM(1,1), NGBM(1,1) and FANGBM(1,1) for the prediction of the cumulative cases of COVID-19 for Italy, UK and USA are given in Table 3. Due to the optimization of the parameters γ and r , it is obvious that FANGBM(1,1) gives the highest prediction performance with having the highest R² and the lowest MAPE and RMSE values. This result is consistent with the results of other studies on forecasting

of Turkey's electricity generation and installed capacity from total renewable and hydro energy [18] and forecasting of renewable energy consumption of China [25].

As a result, it is obtained that FANGBM(1,1) is successful prediction method with having the lowest MAPE, RMSE and the highest R² values. The reason of this can be explained as both parameters γ and r are optimized in this method. In this way, it can adapt to nonlinear curve behaviour

Fig. 1 presents the prediction and forecasting results of the cumulative cases of COVID-19 for Italy using FANGBM(1,1). The model estimates that the cumulative cases of COVID-19 for Italy will be about 233000 on May 22, 2020. Additionally, it can be said that the cumulative cases will continue to increase until this date. However, the daily rate (%) will decrease from 22/04/2020 to 22/05/2020 and the average daily rate is obtained as 0.80% for this period according to this model. Zhang et al. [27] expected the cumulative cases of COVID-19 in Italy will end on June 01, 2020 with the

Table 2
APE (%) values of grey prediction models for this study.

Date	Italy			UK			USA		
	GM(1,1)	NGBM(1,1)	FANGBM(1,1)	GM(1,1)	NGBM(1,1)	FANGBM(1,1)	GM(1,1)	NGBM(1,1)	FANGBM(1,1)
20.03.2020	43.9523	18.8830	6.5043	265.4429	49.9580	10.9209	537.1083	42.6805	18.7400
21.03.2020	30.6055	13.4841	9.1740	213.1594	44.1748	16.6826	586.0590	12.1494	7.5397
22.03.2020	35.0203	0.0000	0.4458	238.0895	24.2684	0.0112	638.7707	27.4652	42.9688
23.03.2020	26.4609	1.7019	0.0001	222.0663	11.9914	6.2550	283.4671	14.6799	9.5492
24.03.2020	20.9417	3.6069	0.9218	197.1748	3.3879	9.1719	151.1353	30.4863	28.9960
25.03.2020	15.5426	4.0165	1.0408	164.1775	0.0000	7.6105	170.4307	9.6232	9.9095
26.03.2020	11.0821	4.0305	1.1536	141.7673	4.5848	8.6066	121.2340	12.9654	14.5778
27.03.2020	6.0636	2.5280	0.0000	113.3636	3.7643	5.0603	138.2319	7.9586	4.9784
28.03.2020	2.0949	1.2350	0.8469	84.6657	0.4839	0.9679	105.6851	5.3700	1.9624
29.03.2020	1.2727	0.0908	1.6866	69.6710	0.0258	1.5520	82.7026	4.0837	0.5395
30.03.2020	3.3862	0.6399	1.7583	60.3536	2.2318	0.0000	65.7308	3.4852	0.0001
31.03.2020	4.0964	0.1240	0.7890	52.6450	4.1591	1.6346	55.6399	5.1603	1.8063
1.04.2020	4.6530	0.2441	0.0098	45.0833	4.9721	2.4602	44.4310	4.3905	1.3423
2.04.2020	5.6929	0.1673	0.0072	33.6573	1.6707	0.5300	35.5139	3.6988	1.0062
3.04.2020	6.4543	0.5225	0.0151	26.1373	0.0973	1.7032	27.9597	2.7082	0.4009
4.04.2020	6.9926	0.8470	0.0424	20.3035	1.1120	2.4411	21.7701	1.6583	0.2638
5.04.2020	7.5557	1.3789	0.3308	18.3049	0.0728	0.7685	15.7510	0.2643	1.8045
6.04.2020	7.6297	1.5489	0.3052	11.9533	3.1321	3.4437	11.0532	1.9338	3.1284
7.04.2020	7.1002	1.2200	0.1759	11.9621	1.4577	1.2743	10.0947	1.0063	1.9175
8.04.2020	6.1126	0.5381	0.9668	12.9242	0.5775	1.2421	8.9243	0.8642	1.5150
9.04.2020	5.6095	0.4931	1.0638	10.8922	0.5308	0.5481	7.8782	1.1644	1.5888
10.04.2020	5.2752	0.7362	0.8213	11.7291	0.4804	1.9315	7.7495	1.1321	1.3703
11.04.2020	4.6950	0.8231	0.6927	11.7068	0.3008	2.0349	7.1273	2.0185	2.1084
12.04.2020	4.5107	1.4134	0.0107	7.2885	4.1957	2.3451	7.9608	2.0058	1.9874
13.04.2020	3.8664	1.6128	0.3147	8.5623	3.9445	1.9778	9.2444	1.9972	1.9071
14.04.2020	2.5813	1.2437	0.1031	11.4626	2.6168	0.5971	11.4126	1.5966	1.4694
15.04.2020	1.1304	0.7963	0.1494	13.6037	2.3106	0.3462	14.9010	0.4427	0.3102
16.04.2020	0.5611	0.1972	0.5179	16.9149	1.3506	0.4834	18.4442	0.3392	0.4456
17.04.2020	1.6305	0.2965	0.1485	20.5696	0.4648	1.1488	21.7575	0.5269	0.5780
18.04.2020	2.9375	0.2355	0.0909	23.4630	0.5507	0.7430	24.6983	0.0412	0.0117
19.04.2020	4.3053	0.1877	0.3733	26.8442	0.5654	0.3323	28.4814	0.1223	0.2544
20.04.2020	5.9952	0.0952	0.4498	30.2700	0.8601	0.4317	32.9512	0.1207	0.3746
21.04.2020	8.2140	0.7967	0.1396	35.3689	0.2190	0.3239	37.8937	0.1400	0.5320
22.04.2020	10.2123	1.2112	0.1427	41.2745	0.6370	0.0704	43.5890	0.0000	0.5444

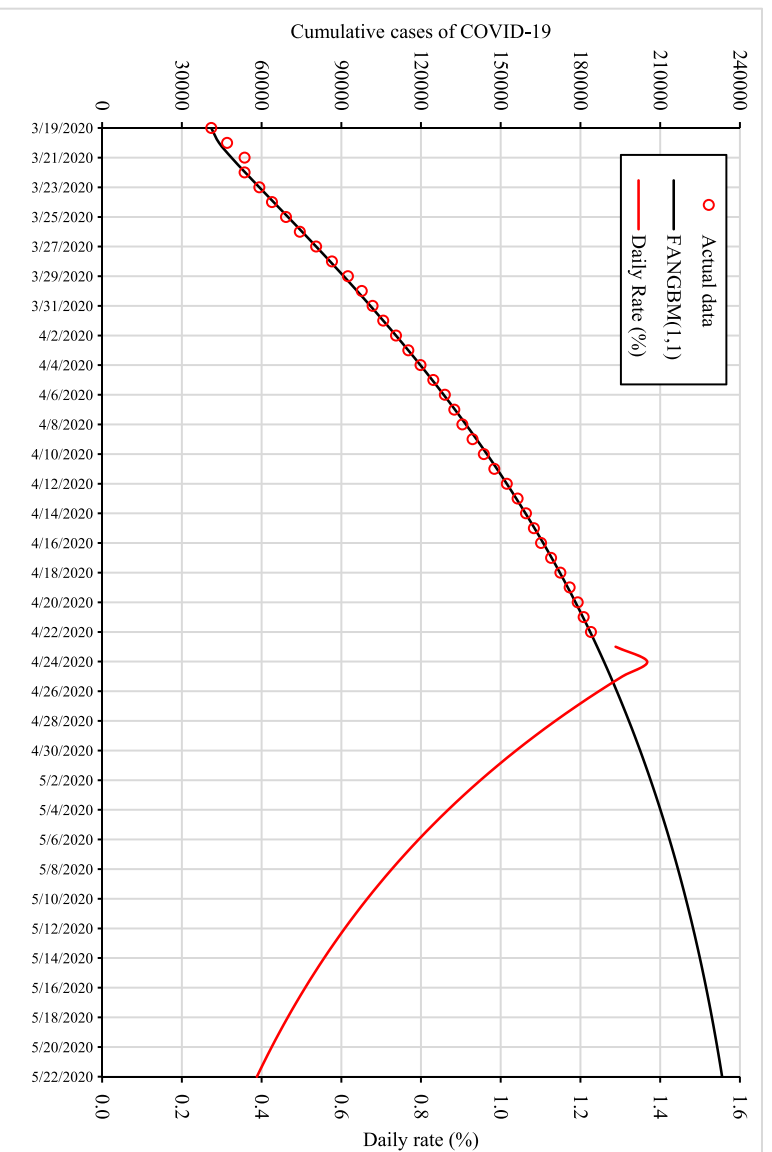


Fig. 1. Prediction and forecasting results of the cumulative cases of COVID-19 for Italy using FANGBM(1,1).

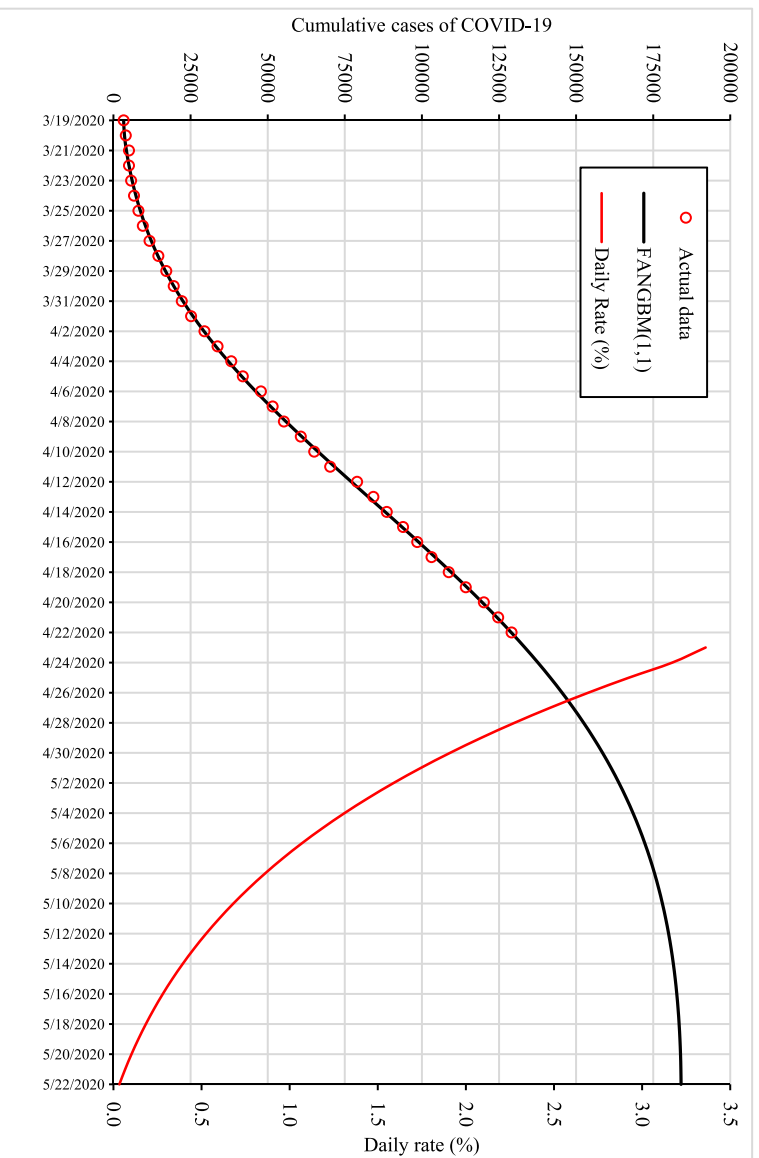


Fig. 2. Prediction and forecasting results of the cumulative cases of COVID-19 for the United Kingdom using FANGBM(1,1).

cumulative cases about 172500. According to the Fig. 1, it can be said that the cumulative cases of COVID-19 in Italy will continue to increase after the date of May 22, 2020.

Forecasting results of the cumulative cases number of COVID-19 for the United Kingdom (UK) are presented in Fig. 2. It can be

said that the cumulative cases of COVID-19 in UK will increase at a diminishing rate from 22/04/2020 to 22/05/2020. Additionally, the cumulative cases will reach to about 189000 on May 22, 2020 with the average daily rate is 1.19% from 22/04/2020 to 22/05/2020. The cases of COVID-19 in UK are expected to end in the early June

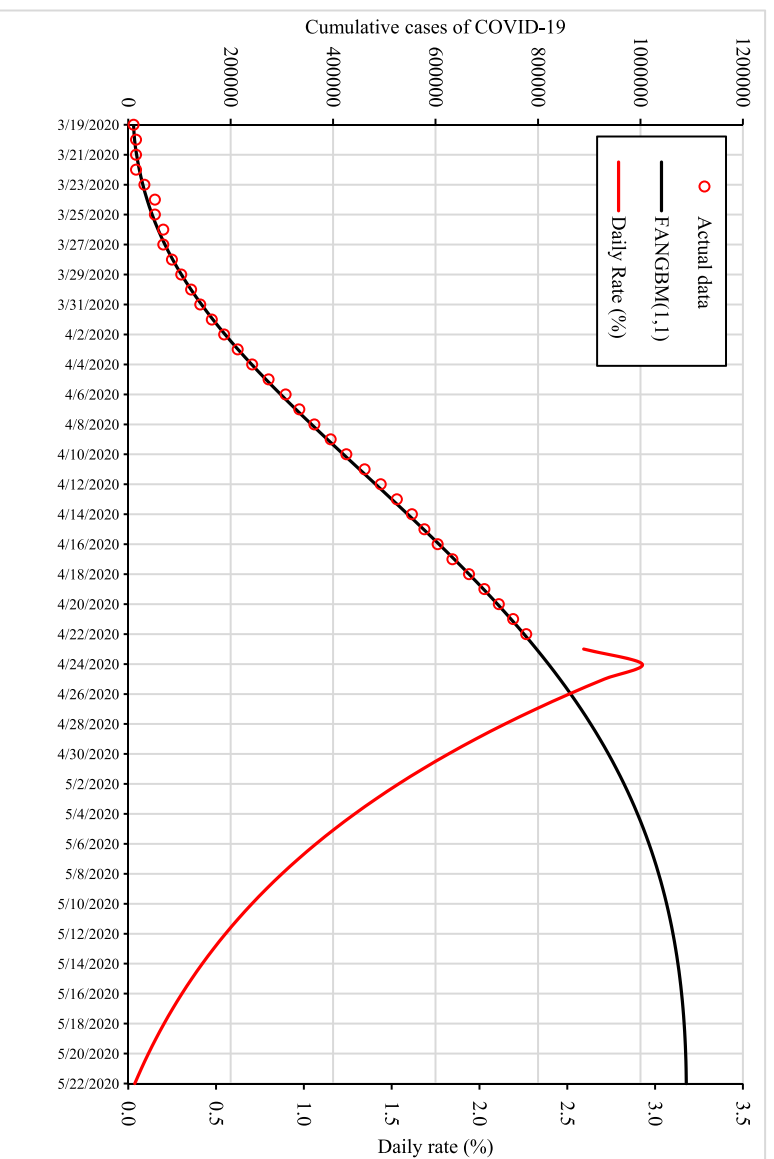


Fig. 3. Prediction and forecasting results of the cumulative cases of COVID-19 for the United States of America using FANGBM(1,1).

(June, 05) with the cumulative cases about 133000 by other study [27]. According to this study [27], the turning point of cases in UK will be April 09, 2020 with the daily new cases are less than 5000. Unfortunately, the number of daily new cases has continued to increase since this date. The daily new cases in UK are reported by World Health Organization (WHO) as more than 8500 on 12/04/2020 and more than 5500 from 18/04/2020 to 20/04/2020.

Fig. 3 gives the prediction and forecasting results of FANGBM(1,1) for the cumulative cases of COVID-19 in USA. It is forecasted that the cumulative cases of COVID-19 for USA will increase at a diminishing rate until May 22, 2020 which correspond to about 1160000. However, the daily rate (%) will decrease from 22/04/2020 to 22/05/2020 with the average daily rate is 1.13% for this period. Zhang et al. [27] forecasted the cases of COVID-19 in USA will end in the early June (June, 03) with the cumulative cases about 835000. In this regard, it can be said that there are similar results between the present study and this study.

5. Conclusions

This study presents forecasting results of the cumulative cases number of coronavirus (COVID-19) in Italy, the United Kingdom (UK) and the United States of America (USA). The GM(1,1), NGBM(1,1) and FANGBM(1,1) models are used to predict the data from 19/03/2020 to 22/04/2020. The prediction performance of the models is tested by mean absolute percentage error (MAPE) and root mean square error (RMSE) and R^2 values. The forecasting procedure is applied from 23/04/2020 to 22/05/2020 using the model which gives the highest prediction performance.

The main conclusion of this study can be given as:

- FANGBM(1,1) is selected as the best prediction model with having the lowest MAPE and RMSE values and the highest R^2 value for all cases.

- The cumulative number of cases of COVID-19 in Italy, UK and USA is forecasted as approximately 233000, 189000 and 1160000, respectively, on 22/05/2020 by using FANGBM(1,1).
- It is estimated that the cumulative cases number of COVID-19 will increase at a diminishing rate until May 22, 2020 for Italy, UK and USA.
- It is forecasted that the daily rate of this outbreak in these countries will decrease until 22/05/2020 which means that the number of daily cases will decrease. However, the rate of decline of daily cases in Italy is expected to be slower than UK and USA until May 22, 2020.
- The highest and the lowest average daily rate of the cumulative cases number are obtained for the United Kingdom and Italy, respectively, from 22/04/2020 to 22/05/2020.

Moreover, following suggestions can be given for further studies on this topic.

- To obtain higher prediction results, the cases of COVID-19 can be forecasted using a hybrid model consisting of decomposition method, deep learning and meta heuristic based optimization technique [2], improved artificial neural network model [9] or machine learning systems [10].
- The optimal parameters r and γ of fractional grey prediction model can be optimized by using other algorithm techniques such as particle swarm optimization (PSO) [28] or grey wolf optimizer (GWO) [12]. By this way, effects of algorithm techniques on the prediction performance can be compared and forecasting results can be diversified.

From politicians and policy makers to business executives, many authorities need more data and information to shape the future of their countries and the world. It is very important that the policies developed through data obtained by scientific method provide a more realistic infrastructure in creating current action plans. Because, human and human health take the crucial role in the

pandemic case which is analysed thoroughly in this study. Since each mistake threatens human health directly, there should be no major inaccuracy in policymaking in this regard. It is expected that the results of this study will help the authorities in developing policies for different fields.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRedit authorship contribution statement

Utkucan Şahin: Software, Methodology, Formal analysis, Data curation. **Tezcan Şahin:** Conceptualization, Validation, Investigation, Writing - original draft, Writing - review & editing.

Acknowledgement

The authors state that there is no conflict of interest. The authors would like to thank the anonymous referee for their valuable comments and suggestions.

References

- [1] Al-qaness MAA, Ewees AA, Fan H, Abd El Aziz M. Optimization method for forecasting confirmed cases of COVID-19 in China. *J Clin Med* 2020;9(3):674.
- [2] Altan A, Karasu S, Bekiros S. Digital currency forecasting with chaotic meta-heuristic bio-inspired signal processing techniques. *Chaos Solitons Fractals* 2019;126:325–36.
- [3] Benvenuto D, Giovanetti M, Vassallo L, Angeletti S, Ciccozzi M. Application of the ARIMA model on the COVID-2019 epidemic dataset. *Data in Brief* 2020;29:105340.
- [4] Chen C. Application of the novel nonlinear grey Bernoulli model for forecasting unemployment rate. *Chaos Solitons Fractals* 2008;37(1):278–87.
- [5] Deng J. Control problems of grey systems. *Systems and Control Lett* 1982;1(5):288–94.
- [6] Ding S, Xu N, Ye J, Zhou W, Zhang X. Estimating Chinese energy-related CO₂ emissions by employing a novel discrete grey prediction model. *J Cleaner Prod* 2020;259:120793.
- [7] Fanelli D, Piazza F. Analysis and forecast of COVID-19 spreading in China, Italy and France. *Chaos Solitons Fractals* 2020;134:109761.
- [8] Gao M, Mao S, Yan X, Wen J. Estimation of Chinese CO₂ emission based on a discrete fractional accumulation grey model. *The Journal of Grey System* 2015;27(4):114–30.
- [9] Jun W, Yuyan L, Lingyu T, Peng G. Modeling a combined forecast algorithm based on sequence patterns and near characteristics: an application for tourism demand forecasting. *Chaos Solitons Fractals* 2018;108:136–47.
- [10] Lahmiri S, Bekiros S. Intelligent forecasting with machine learning trading systems in chaotic intraday Bitcoin market. *Chaos Solitons Fractals* 2020;133:109641.
- [11] Lewis CD. *Industrial and Business Forecasting Method*. London: Butterworth-Heinemann; 1982.
- [12] Ma X, Xie M, Wu W, Zeng B, Wang Y, Wu X. The novel fractional discrete multivariate grey system model and its applications. *Appl Math Modell* 2019;70:402–24.
- [13] Magal P, Webb G. Predicting the number of reported and unreported cases for the COVID-19 epidemic in South Korea, Italy, France and Germany. SSRN 2020. <http://dx.doi.org/10.2139/ssrn.3557360>.
- [14] Mao S, Gao M, Xiao X, Zhu M. A novel fractional grey system model and its application. *Appl Math Modell* 2016;40:5063–76.
- [15] Microsoft Corporation. Microsoft Excel 2016 MSO (for Windows) [computer program]. WA, USA: Version 16.0.4266.1001, Redmond; 2016.
- [16] Petropoulos F, Makridakis S. Forecasting the novel coronavirus COVID-19. *PLoS One* 2020;15(3):e0231236.
- [17] Roosa K, Lee Y, Luo R, Kirpich A, Rothenberg R, Hyman JM, Yan P, Chowell G. Short-term Forecasts of the COVID-19 Epidemic in Guangdong and Zhejiang, China: February 13–23. *J Clin Med* 2020;9(2):596.
- [18] Şahin U. Projections of Turkey's electricity generation and installed capacity from total renewable and hydro energy using fractional nonlinear grey Bernoulli model and its reduced forms. *Sustainable Production and Consumption* 2020;23:52–62. doi:10.1016/j.spc.2020.04.004.
- [19] Wang Q, Song X, Li R. A novel hybridization of nonlinear grey model and linear ARIMA residual correction for forecasting U.S. shale oil production. *Energy* 2018;165:1320–31.
- [20] Wang Z-X, Li Q. Modelling the nonlinear relationship between CO₂ emissions and economic growth using a PSO algorithm-based grey Verhulst model. *J Cleaner Prod* 2019;207:214–24.
- [21] WHO, 2020a. World Health Organization. <<https://www.who.int/emergencies/diseases/novel-coronavirus-2019/events-as-they-happen>>[Accessed April 18, 2020].
- [22] WHO, 2020b. World Health Organization. <<https://covid19.who.int/>>[Accessed April 18, 2020].
- [23] Wu L. Using fractional GM(1,1) model to predict the life of complex equipment. *Grey Systems: Theory and Application* 2016;6(1):32–40.
- [24] Wu L, Liu S, Yao L, Yan S, Liu D. Grey system model with the fractional order accumulation. *Communications Nonlinear Science and Numerical Simulation* 2013;18:1775–85.
- [25] Wu W, Ma X, Zeng B, Wang Y, Cai W. Forecasting short-term renewable energy consumption of China using a novel fractional nonlinear grey Bernoulli model. *Renewable Energy* 2019;140:70–87.
- [26] Xu N, Ding S, Gong Y, Bai J. Forecasting Chinese greenhouse gas emissions from energy consumption using a novel grey rolling model. *Energy* 2019;175:218–27.
- [27] Zhang X, Ma R, Wang L. Predicting turning point, duration and attack rate of COVID-19 outbreaks in major Western countries. *Chaos Solitons Fractals* 2020;135:109829.
- [28] Zhu, X., Dang, Y., Ding, S., 2020. Using a self-adaptive grey fractional weighted model to forecast Jiangsu's electricity consumption in China, *Energy*, 190, 116417.