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Performance of maximum EWMA control chart in the presence of measurement error using auxiliary information

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ABSTRACT

EWMA and Max-EWMA charts are considered efficient for individual as well as joint monitoring of mean and variance shifts in the production process. However, measurement error is affecting the efficiency of these charts. In this study we propose a maximum exponentially weighted moving average with measurement error using auxiliary information control chart and name it Max-EWMAMEAI control chart. The efficiency of this chart is highlighted and the effect of measurement error is shown by the values of *ARLs* and *SDRLs* calculated through simulations using linear covariate model. The case of linearly increasing variance is examined and multiple measurements technique has been applied to reduce the error effect. A real life example is also included to support the simulation results.

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Auxiliary information; Max-EWMAMEAI control chart; Measurement error; Quality characteristic; Statistical process control

1. Introduction

Statistical process control (SPC) provides a number of control charts for monitoring shifts in location as well as scale parameters of quality characteristics for different industrial production processes. Shewhart (1925) presented the concept of control charts and is considered as a pioneer of control charts applicable for industrial production. Control charts by Shewhart like $\bar{X}$, S^2, S etc. are memoryless and depend upon the value of last observation or sample but do not use the past information of the production process. Later on, researchers used past information on the production process and improved efficiency of their control charts as compared with those of Shewhart's. The improved control charts include exponentially weighted moving average (EWMA) control charts developed by Roberts (1959), cumulative sum (CUSUM) control charts by Page (1954) which are called memory based control charts. Memory based control charts have been further improved and developed for joint monitoring of process mean as well as variance rather to monitor mean and variance independently by applying two different charts for each of them. These include the max-EWMA chart developed by Xie (1999) to monitor mean and variance jointly. In addition to others, Khoo, Teh, and Wu (2010), Abbas, Riaz, and Does (2011), Chowdhury, Mukherjee, and Chakraborti

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(2015), Dogu (2015), Lu and Huang (2017), Haq, Gulzar, and Khoo (2018), Tayyab, Noor-Ul-Amin, and Hanif (2019) also contributed to develop EWMA and CUSUM charts in one or the other way. However, researchers could not discuss the errors during the measurement of quality characteristics and this issue has not been discussed during the establishment of their control charts. It has been considered understood that all the variables for which data are collected to prepare the control charts are measured correctly. But it needs to be investigated whether the control charts prepared with the assumption of accurate measurements, can lead to misleading results and conclusions or not? Therefore, we highlight this issue of measurement error in this paper for the development of control charts for joint monitoring of the process mean and variance.

Measurement error refers to the difference between actual and collected or measured values of the quality characteristic for which we prepare control charts to monitor any shift in the location or dispersion parameters. Measurement error effects control charts negatively and destroys their efficiency by proceeding to misleading decisions. Montgomery (2009) highlighted that for effective control charts, accurate measurements are necessary. Montgomery and Runger (1993) said that quality of the product completely depends upon the accurate measurements of the sample observations. Sanders (1995) revealed that disturbances and measurement errors affect the control system of chemical production process. Mittag and Stemann (1998) uncovered that measurement error reduces the efficiency of mean and variance charts developed by Shewhart. Linna and Woodall (2001) elaborated the measurement error model and proved that error has negative effect on the chart efficiency. Maravelakis, Panaretos, and Psarakis (2004) used covariate model to study the error impact and proved that measurement error affects the efficiency of EWMA chart. They improved the chart efficiency by applying multiple measurements technique. Huwang and Hung (2007) proved that impact of measurement error is negative for chart efficiency. Li and Huang (2009) uncovered that measurement error has negative impact on performance of regression-based multivariate process monitoring. Wu (2011) said that if measurement error is not considered, it may lead to the undesired and unreliable decisions for quality control process. Maravelakis (2012) studied the impact of measurement error on the efficiency of CUSUM control chart for mean and used multiple measurements method to reduce the error effect. Baral and Anis (2015) analyzed that ignoring measurement error can drive to misleading results for process control. Haq et al. (2015) used ranked set sampling schemes to study the error effect and concluded that EWMA charts based on median ranked set sampling and imperfect MRSS perform better than counterparts in the presence of measurement error. Hu et al. (2015) analyzed that measurement error is always existed in the quality control applications while they used linearly covariate model to study the error impact. Abbasi (2016) highlighted impact of measurement error on EWMA chart using Monte Carlo simulations and used multiple measurements to reduce the error effect. Dizabadi, Shahrokhi, and Maleki (2016) studied the adverse error effect for joint monitoring of mean and variance using values of *ARLs* and *SDRLs*. Hu et al. (2016) studied the effects of measurement error on the performance of mean chart using linear covariate model. Maleki, Amiri, and Ghashghaei et al. (2016) focused on the measurement error existed in the system and its impact on the joint monitoring through *ARLs* and *SDRLs* values. Ghashghaei et al. (2016) used ranked set sampling to study the error impact. They proved that RSS is better than SRS for shift detection during joint monitoring even in the presence of measurement error. Tran, Castagliola, and Celano (2016) studied that measurement error has negative impact on the performance of Shewhart-RZ chart. Daryabari et al. (2017) investigated the measurement error effect on the maximum EWMA and mean squared deviation charts using linear covariate error model. They proved that measurement error negatively affect the chart performance for joint monitoring using average time to signal. Maleki and Salmasnia (2017) used combination of CUSUM and generalized likelihood ratio procedures to analyze the effect of measurement error. Maleki, Amiri, and Castagliola (2017) focused two sources of error i.e., operator's ignorance and measurement error. They highlighted three error models i.e., additive model, multiplicative model and two-component measurement error model, to study the relationship between observed and actual sampled values. Sabahno and Amiri (2017) proved that up to four multiple measurements can reduce the measurement error effect but after that result cannot further improve. They used variable sample size and sampling interval control charts to study the measurement error effect during monitoring mean process shift. Unnati and Raj (2017) also tried to study the impact of measurement error on the performance of mean chart and on the power of the chart to detect process shift.

In spite of previous work on control charts affected by measurement error, a few authors in recent years also tried to study the impact of measurement error on the efficiency of control charts for process monitoring. Amiri, Ghashghaei, and Maleki (2018) analyzed the measurement error effect for joint monitoring in multivariate production process using mean vector and covariance matrix. They used four techniques to reduce the error effect i.e., more samples, multiple measurements of the same samples, excluding outliers, and to omit the outliers. Cheng and Wang (2018) used linear covariate model to study the measurement error impact on the performance of EWMA median and CUSUM median control charts through values of ARLs and SDRLs. Salmasnia, Maleki, and Niaki (2018) proved that ranked set sampling and large samples are two remedial measures to reduce the measurement error impact on the efficiency of Max-EWMA control chart for joint monitoring. Tang et al. (2018) focused on the performance of adaptive EWMA chart for mean monitoring, affected by measurement error. They proved that proposed adaptive EWMA chart is superior to usual EWMA chart even in the presence of measurement error. Riaz et al. (2019) proposed mixed EWMA-CUSUM chart using regression estimator for monitoring process mean. They concluded that this chart is better than other existing charts in the presence of measurement error. Noor-Ul-Amin, Riaz, and Safeer (2019) examined the measurement error impact on the efficiency of auxiliary information based EWMA chart for monitoring of shift in the process mean using covariate method, multiple measurements technique and linearly increasing variance method. Asif, Khan, and Noor-Ul-Amin (2020) highlighted the impact of measurement error in the performance of hybrid EWMA chart. For research work, many techniques have been adopted by the authors to increase the efficiency of control charts. One of the techniques is to use more information for the development of any statistic to be utilized in a control chart. It has been observed that as we use more information in a control chart, its efficiency is improved e.g., memory less and memory based control charts. Having this basic idea into consideration, the use of auxiliary information enters in fashion to improve the efficiency of a control chart.

Auxiliary information is some additional or prior information for a variable other than the study variable, available from records or is collected without additional cost or with very less cost and time. However, there must be a strong correlation between study and auxiliary variables to get benefit from auxiliary variable. Use of auxiliary variable(s) in survey sampling is very popular to increase the efficiency of estimator(s), e.g. Kadilar and Cingi (2006), Singh and Solanki (2012), Noor-Ul-Amin, Javaid, and Hanif (2017), Javaid, Noor-Ul-Amin, and Hanif (2019) and many others used auxiliary information to increase the efficiency of their proposed estimators. As the use of auxiliary information proved efficiency in survey sampling, therefore, authors have also started the use of auxiliary information for the development of efficient control charts. Riaz (2008), Riaz et al. (2013), Abbas, Riaz, and Does (2014), Haq and Khoo (2016), Sanusi, Abbas, and Riaz (2017), Sanusi et al. (2017), Javaid, Noor-ul-Amin, and Hanif (2020), Noor-Ul-Amin, Khan, and Sanaullah (2019) and others used auxiliary information to develop efficient control charts. It has been observed by authors that the inclusion of auxiliary information improves the efficiency of control charts. In this article, we also try to propose a control chart using auxiliary information to increase the efficiency and how this efficiency is affected by the measurement error. We name it maximum exponentially weighted moving average with measurement error using auxiliary information (Max-EWMAMEAI) control chart.

The sequence of this article is that after introduction, Sec. 2 comprises proposed Max-EWMAMEAI control chart, Sec. 3 elaborates impact of measurement error on the proposed chart through calculations of *ARLs*, *SDRLs* and their explanation through graphs, Sec. 4 is the real life example, main findings are in Sec. 5, while conclusion is drawn in Sec. 6.

2. Proposed Max-EWMA with measurement error using auxiliary information (Max-EWMAMEAI) control chart

The usual additive error model is $Y = X + \varepsilon$, which was introduced by Bennet (1954) to study the variable of interest Y. If actual Y cannot be obtained due to error in taking the measurement, then X is the true value of the study variable and ε is the random error due to measurement issue. For this model, it is assumed that $Y \sim N(\mu, \sigma^2)$, where μ is the mean and σ^2 is the variance i.e., $\sigma^2 = \sigma_p^2 + \sigma_m^2$, whereas, σ_p^2 is the variance of true value and σ_m^2 is the variance of error component.

Let X be a normally distributed variable and its actual value cannot be measured correctly. However, its related value Y which is covariate of X, can be obtained by using a linear covariate model $Y = A + BX + \varepsilon$, where A and B are constants while ε is random error term which is independent of X and is normally distributed with constant variance as $\varepsilon \sim N(0, \sigma_m^2)$. If $A = 0$ and $B = 1$, the covariate model will become the usual additive error model. For the in-control production process, the quality characteristic X is normally distributed, so its covariate Y is also normally distributed i.e., $Y \sim N((A + B\mu_x), (B^2\sigma_p^2 + \sigma_m^2))$, where $\mu_y = A + B\mu_x$ is the mean and $\sigma_y^2 = B^2\sigma_p^2 + \sigma_m^2$ is the variance.

If process mean shifts from μ_x to $\mu_x + \gamma\sigma_p$, it will also change mean of the covariate Y as $A + B(\mu_x + \gamma\sigma_p)$. On the other hand, if variance of process shifts from σ_p^2 to $\delta^2\sigma_p^2$, such that $\delta > 0$, it will also change the variance of covariate Y as $B^2\delta^2\sigma_p^2 + \sigma_m^2$. For an in-control production process, there is no shift in mean as well as variance i.e., $\gamma = 0$ and $\delta = 1$, thus shift in mean and variability is expected. To study the effects of process shifts and measurement error, let us discuss our proposed control chart.

The use of EWMA statistic is common for joint monitoring of process mean and / or variance during the production process, particularly for moderate and small shifts. To increase the efficiency of this chart for joint monitoring of moderate and small shifts, auxiliary information can be utilized. However, if the effect of measurement error is not considered for this chart, its efficiency may suffer to some extent which could be harmful to the production process and ultimately for the profit of that industrial production unit. Therefore, in this paper we study the effect of measurement error and propose maximum exponentially weighted moving average with measurement error using auxiliary information (Max-EWMAMEAI) control chart.

If Y is quality characteristic of a variable of production process and W is considered as an auxiliary variable, while there is a strong correlation between both the variables. We select a sample from both the variables, say (Y_{ij}, W_{ij}) is the j^{th} sample of size n, and i=1, 2, 3,….n while $j = 1, 2, 3, \ldots\ldots$ Both the variables are normally distributed and have bivariate normal distribution as $(Y, W) \sim N_2(\mu_y, \mu_w, \sigma_y^2, \sigma_w^2, \rho)$, where N_2 stands for bivariate normal distribution, μ_y & σ_y^2 are the mean and variance of Y, μ_w & σ_w^2 are the mean and variance of auxiliary variable W, whereas, ρ is the correlation coefficient between Y and W variables. It is also assumed that population parameters $\mu_y, \mu_w, \sigma_y^2, \sigma_w^2, \rho$ are known. The sample statistics for mean and variance for the j^{th} sample are:

$$\bar{y}_j = \frac{\sum_{i=1}^{n} y_{ij}}{n}, \bar{w}_j = \frac{\sum_{i=1}^{n} w_{ij}}{n}, S_{y_j}^2 = \frac{\sum_{i=1}^{n} \left(y_{ij} - \bar{y}_j\right)^2}{n-1} \text{ and } S_{w_j}^2 = \frac{\sum_{i=1}^{n} \left(w_{ij} - \bar{w}_j\right)^2}{n-1}.$$

We assume that the production process is in-control and there is no shift in mean or variance of the variable of interest. Keeping in view the above sample statistics, the regression estimator of mean using auxiliary variable can be written as:

$$U_j = \bar{y}_j + \beta(\mu_w - \bar{w}_j), \tag{1}$$

where $\beta = \rho\left(\frac{\sigma_y}{\sigma_w}\right)$, $E(U_j) = \mu_y = A + B\mu$ is the mean, and $Var(U_j) = \frac{\sigma_y^2(1-\rho^2)}{n} = \frac{(B^2\sigma_p^2+\sigma_m^2)(1-\rho^2)}{n}$ is the variance under the covariate model. For the covariate model, if we apply transformation on Eq. (1), we can write the transformed estimator for mean as:

$$U_{je} = \frac{U_j - E(U_j)}{Var(U_j)} = \frac{\bar{Y} - (A + B\mu)}{\sqrt{\left\{(B^2\sigma_p^2 + \sigma_m^2)(1-\rho^2)\right\}/n}}, \tag{2}$$

which follows the standard normal distribution as $U_{je} \sim N(0,1)$, whereas, $\frac{(n-1)S_{w_j}^2}{\sigma_w^2} \sim \chi^2_{(n-1)}$, $\frac{(n-1)S_{y_j}^2}{B^2\sigma_p^2+\sigma_m^2} \sim \chi^2_{(n-1)}$, $Var_{(w_j)} = \phi^{-1}\left[H\left\{\frac{(n-1)S_{w_j}^2}{\sigma_w^2}, (n-1)\right\}\right] \sim N(0,1)$, and

$Var_{(y_j)} = \phi^{-1}\left[H\left\{\frac{(n-1)S^2_{y_j}}{B^2\sigma^2_p+\sigma^2_m},(n-1)\right\}\right] \sim N(0,1)$ for in-control production process, where $H(\psi,\upsilon)$ follow the chi-square distribution with υ degrees of freedom, and ϕ^{-1} is the inverse of the standard normal distribution function.

Difference estimator for variance of the in-control production process, using variances of study and auxiliary variables, can be written as:

$$V_j = Var_{(y_j)} - \rho^* Var_{(w_j)}, \tag{3}$$

where ρ^* is the correlation coefficient between $Var_{(y_j)}$ and $Var_{(w_j)}$. The mean and variance of V_j are: $E(V_j) = 0$, and $Var(V_j) = 1-\rho^{*2}$. By applying transformation on Eq. (3) using covariate model, we can write an estimator for the variance of the production process as:

$$V_{je} = \frac{V_j - E(V_j)}{\sqrt{(1-\rho^{*2})}}, \tag{4}$$

whereas, $V_{je} \sim N(0,1)$ for in-control production process.

By taking advantage of transformed statistics (2) and (4), we can easily state EWMA with measurement error using auxiliary information, statistics for mean and variance respectively as:

$$Z_{mj} = \lambda U_{je} + (1-\lambda)Z_{m(j-1)} \tag{5}$$

$$Z_{vj} = \lambda V_{je} + (1-\lambda)Z_{v(j-1)} \tag{6}$$

where Z_{mj} is for mean and Z_{vj} is for variance, while both are independent due to mutual independence of U_{je} and V_{je}. Both the statistics are normally distributed as $Z_{mj} \sim N(0,\sigma^2_{Z_m})$ and $Z_{vj} \sim N(0,\sigma^2_{Z_v})$ for in-control production process.

Our proposed final maximum exponentially weighted moving average with measurement error using auxiliary information (Max-EWMAMEAI) control chart can be presented using equations (5) and (6). The plotting Max-EWMAMEAI statistic can be written as:

$$Max - EWMAMEAI = Mx = Max\left(|Z_{mj}|, |Z_{vj}|\right). \tag{7}$$

As plotting statistic is the maximum of absolute values of mean and variance statistics, so values of statistics for all the samples will be plotted against upper control limit (UCL) which is given as:

$$UCL = E(Mx) + L\sqrt{Var(Mx)}, \tag{8}$$

For Maximum EWMA control chart, Xie (1999) calculated the UCL for thej^{th} sample for joint monitoring as:

$$UCL_j = (1.128379 + 0.602810 \times L)\sqrt{Var(Mx)}, \tag{9}$$

where $Var(Mx) = \frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)^{2j}\right\}$ which becomes $\frac{\lambda}{2-\lambda}$ for largely repetitive samples.

Therefore, for large samples, the expression (9) can finally be written as:

$$UCL = (1.128379 + 0.602810 \times \mathrm{L})\sqrt{\frac{\lambda}{(2-\lambda)}}, \tag{10}$$

where λ is smoothing constant and $0 < \lambda \leq 1$, L is the width of the control limit and by using it we can arrive at the desired average run length for in-control process i.e., ARL_0, whereas, $Var(Z_{mj}) = Var(Z_{vj}) = \frac{\lambda}{2-\lambda}$ for large samples.

3. Impact of measurement error on proposed Max-EWMAMEAI control chart

Let us investigate the effect of measurement error for our proposed control chart through average run lengths ($ARLs$) and standard deviation of run lengths ($SDRLs$) using Monte Carlo simulations for 50,000 replicates, presented in Table 1 and first four graphs. When a production process is running smoothly and if some variation is occurred due to assignable cause, it is referred to as process shift, which may be in mean or variance or both for joint monitoring. The production process goes out of control when a shift takes place. The performance of a control chart can be judged from its power of detection of this shift at the earliest. Average run length (ARL) is considered as the best technique to study the performance of control charts and Montgomery (2009) referred ARL as the average number of observations or samples remaining within the control limits of a chart until the first out of control observation or sample in a production process. So, if we plot observations or sample statistics on a control chart, the number of samples until the first observation or statistic falls out of control limits of the chart, is called run length i.e., RL. If we replicate this process for a large number of times, it will generate the run length distribution. The average of run length distribution will become ARL and its standard deviation as $SDRL$. Let us do this exercise in two ways (a) when the population parameters are known and (b) when population parameters are unknown.

3.1. Known population parameters

For an in-control production process, it is assumed that the study and auxiliary variables follow the bivariate normal distribution i.e., $(Y, W) \sim N_2(\mu_y, \mu_w, \sigma_y^2, \sigma_w^2, \rho)$, where $\mu_y = \mu_w = 0$, $\sigma_y^2 = \sigma_w^2 = 1$ and ρ has different known values like 0.00, 0.25, 0.50, 0.95. We calculated $ARLs$ and $SDRLs$ using these known parameters as shown in Table 1 and Figures 1–4.

We utilize the $ARLs$ and $SDRLs$ to investigate the efficiency of the proposed control chart, the effect of measurement error on the efficiency and multiple measurements technique to reduce the effect of measurement error using covariate model. Monte Carlo simulation method is used for calculations of $ARLs$ and $SDRLs$ using R-Language. Different combinations of shifts in mean (γ) and variance (δ) are arranged for calculations which are shown in Table 1. Correlation coefficients between study and auxiliary variables (ρ) and between their variances (ρ^*) are used as $(\rho, \rho^*) = (0, 0)$, (0.25, 0.05), (0.50, 0.23), (0.95, 0.89) for this study. Different values, from 0.0 (no correlation) to

Table 1. *ARLs* and *SDRLs* of Max-EWMAMEAI chart with different values of $\sigma_m^2 \div \sigma^2$.

			$\sigma_m^2 \div \sigma^2$											
			No error		0.1		0.2		0.3		0.5		1	
δ	ρ	γ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.25	0.00	0.00	4.16	0.49	4.31	0.54	4.75	0.64	5.40	0.81	7.34	1.45	17.34	6.53
		0.10	4.16	0.48	4.31	0.54	4.75	0.65	5.40	0.81	7.34	1.45	17.26	6.52
		0.25	4.16	0.49	4.31	0.54	4.75	0.65	5.40	0.81	7.33	1.45	17.00	6.18
		0.50	4.15	0.49	4.31	0.54	4.75	0.64	5.40	0.81	7.32	1.39	13.67	3.81
		1.00	4.13	0.46	4.26	0.50	4.58	0.52	4.86	0.49	5.36	0.67	7.03	1.42
		2.00	2.55	0.48	2.56	0.47	2.60	0.43	2.66	0.38	2.83	0.25	3.45	0.53
		3.00	1.98	0.00	1.99	0.00	1.99	0.00	2.00	0.00	2.00	0.01	2.35	0.48
	0.25	0.00	4.15	0.48	4.31	0.54	4.75	0.65	5.40	0.80	7.33	1.45	17.25	6.52
		0.10	4.15	0.49	4.31	0.54	4.75	0.64	5.40	0.81	7.33	1.44	17.20	6.42
		0.25	4.15	0.49	4.31	0.54	4.74	0.64	5.39	0.81	7.33	1.45	16.80	6.01
		0.50	4.15	0.49	4.31	0.54	4.74	0.64	5.39	0.79	7.28	1.37	13.35	3.69
		1.00	4.08	0.42	4.20	0.48	4.48	0.52	4.74	0.51	5.19	0.66	6.79	1.36
		2.00	2.35	0.48	2.40	0.49	2.50	0.50	2.61	0.48	2.74	0.34	3.35	0.51
		3.00	1.97	0.00	1.98	0.00	1.98	0.00	2.00	0.00	2.00	0.00	2.25	0.43
	0.50	0.00	4.06	0.47	4.21	0.52	4.63	0.63	5.26	0.78	7.10	1.40	16.61	6.11
		0.10	4.06	0.47	4.21	0.52	4.63	0.63	5.25	0.78	7.10	1.39	16.60	6.07
		0.25	4.06	0.47	4.21	0.52	4.63	0.62	5.25	0.79	7.10	1.39	16.11	5.60
		0.50	4.05	0.47	4.21	0.52	4.63	0.62	5.25	0.79	7.01	1.27	12.13	3.16
		1.00	3.91	0.37	3.97	0.37	4.12	0.43	4.29	0.51	4.66	0.63	6.00	1.14
		2.00	2.04	0.19	2.05	0.21	2.07	0.25	2.13	0.34	2.42	0.49	3.05	0.38
		3.00	1.92	0.11	1.93	0.10	1.95	0.09	1.97	0.05	1.99	0.02	2.04	0.18
	0.95	0.00	2.07	0.25	2.11	0.31	2.26	0.44	2.58	0.53	3.33	0.58	6.59	1.59
		0.10	2.07	0.26	2.10	0.31	2.26	0.44	2.58	0.52	3.32	0.58	6.59	1.56
		0.25	2.07	0.26	2.10	0.30	2.25	0.44	2.56	0.52	3.29	0.56	6.23	1.39
		0.50	2.04	0.25	2.07	0.29	2.19	0.41	2.44	0.51	3.01	0.55	4.27	0.91
		1.00	1.77	0.43	1.78	0.42	1.82	0.42	1.87	0.40	1.98	0.33	2.22	0.44
		2.00	1.00	0.09	1.00	0.09	1.00	0.09	1.00	0.08	1.00	0.09	1.09	0.33
		3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.50	0.00	0.00	7.33	1.45	7.50	1.51	7.96	1.67	8.69	1.97	11.19	3.12	25.69	12.22
		0.10	7.33	1.45	7.49	1.51	7.92	1.68	8.68	1.96	11.17	3.10	25.64	12.15
		0.25	7.33	1.44	7.49	1.50	7.93	1.68	8.67	1.97	11.15	3.05	23.52	10.02
		0.50	7.26	1.34	7.41	1.37	7.75	1.46	8.34	1.63	9.94	2.12	15.16	4.81
		1.00	4.81	0.60	4.83	0.60	4.92	0.64	5.04	0.69	5.43	0.85	7.05	1.55
		2.00	2.55	0.50	2.56	0.49	2.60	0.48	2.66	0.44	2.84	0.34	3.46	0.56
		3.00	1.98	0.00	1.99	0.00	1.99	0.00	2.00	0.00	2.00	0.02	2.36	0.48
	0.25	0.00	7.34	1.45	7.47	1.52	7.91	1.66	8.67	1.94	11.17	3.10	25.65	12.32
		0.10	7.33	1.44	7.47	1.51	7.91	1.65	8.67	1.98	11.15	3.10	25.37	12.07
		0.25	7.33	1.45	7.46	1.47	7.89	1.65	8.67	1.96	11.12	3.02	23.07	9.75
		0.50	7.23	1.30	7.33	1.33	7.68	1.44	8.26	1.58	9.75	2.06	14.63	4.55
		1.00	4.66	0.59	4.69	0.60	4.78	0.63	4.89	0.67	5.25	0.82	6.79	1.47
		2.00	2.41	0.49	2.43	0.50	2.50	0.50	2.61	0.49	2.74	0.39	3.36	0.53
		3.00	1.97	0.01	1.98	0.01	1.98	0.01	2.00	0.00	2.00	0.01	2.26	0.44
	0.50	0.00	7.13	1.38	7.27	1.45	7.70	1.61	8.41	1.90	10.81	2.92	24.61	11.60
		0.10	7.13	1.39	7.26	1.44	7.67	1.61	8.41	1.88	10.79	2.94	24.38	11.32
		0.25	7.13	1.40	7.26	1.45	7.71	1.61	8.41	1.87	10.79	2.91	21.78	8.85
		0.50	6.87	1.17	6.99	1.19	7.30	1.28	7.75	1.40	8.98	1.82	13.02	3.90
		1.00	4.19	0.54	4.21	0.55	4.27	0.57	4.37	0.60	4.69	0.73	6.03	1.23
		2.00	2.06	0.24	2.07	0.25	2.11	0.31	2.17	0.38	2.43	0.50	3.06	0.41
		3.00	1.92	0.17	1.93	0.16	1.95	0.13	1.97	0.10	1.99	0.04	2.05	0.21
	0.95	0.00	3.32	0.57	3.38	0.59	3.53	0.64	3.80	0.71	4.65	0.93	8.96	2.57
		0.10	3.32	0.58	3.37	0.59	3.53	0.63	3.79	0.71	4.65	0.94	8.91	2.52
		0.25	3.28	0.56	3.33	0.57	3.47	0.62	3.75	0.69	4.51	0.87	7.56	1.87
		0.50	2.88	0.56	2.90	0.57	2.98	0.59	3.09	0.63	3.39	0.70	4.33	0.91
		1.00	1.86	0.37	1.87	0.36	1.89	0.34	1.93	0.30	1.99	0.22	2.22	0.42
		2.00	1.00	0.02	1.00	0.02	1.00	0.01	1.00	0.02	1.00	0.03	1.09	0.30
		3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.75	0.00	0.00	19.56	8.03	19.91	8.14	20.85	8.86	22.78	10.16	29.12	14.89	69.82	50.77
		0.10	19.42	7.88	19.79	8.03	20.79	8.58	22.64	10.00	28.66	14.30	63.92	44.77

(*continued*)

Table 1. Continued.

			$\sigma_m^2 \div \sigma^2$											
			No error		0.1		0.2		0.3		0.5		1	
δ	ρ	γ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
		0.25	17.45	6.01	17.67	6.09	18.32	6.46	19.47	7.15	22.80	9.37	37.95	21.21
		0.50	10.28	2.57	10.33	2.59	10.54	2.70	10.87	2.89	11.88	3.44	16.27	6.06
		1.00	4.87	0.85	4.89	0.85	4.96	0.88	5.09	0.92	5.48	1.08	7.05	1.71
		2.00	2.55	0.50	2.56	0.50	2.60	0.49	2.66	0.47	2.84	0.42	3.47	0.60
		3.00	1.98	0.06	1.99	0.04	1.99	0.04	2.00	0.03	2.01	0.08	2.37	0.48
	0.25	0.00	19.50	7.82	19.82	8.15	20.85	8.87	22.78	10.24	29.12	14.91	69.82	50.59
		0.10	19.41	7.80	19.79	8.06	20.79	8.75	22.51	9.82	28.66	14.43	63.16	43.58
		0.25	17.13	5.80	17.41	5.89	18.00	6.25	19.03	6.98	22.37	8.97	36.19	19.97
		0.50	9.93	2.44	9.99	2.48	10.15	2.57	10.54	2.77	11.47	3.28	15.59	5.81
		1.00	4.71	0.81	4.73	0.81	4.81	0.85	4.93	0.89	5.31	1.02	6.83	1.65
		2.00	2.44	0.50	2.46	0.50	2.50	0.50	2.57	0.50	2.74	0.45	3.38	0.57
		3.00	1.97	0.07	1.98	0.07	1.98	0.06	1.99	0.05	2.00	0.05	2.28	0.45
	0.50	0.00	18.80	7.51	18.89	7.63	20.14	8.50	21.87	9.58	27.85	13.98	66.11	48.23
		0.10	18.69	7.37	18.81	7.74	19.86	8.08	21.66	9.29	27.32	13.40	59.25	40.39
		0.25	15.89	5.07	16.12	5.21	16.76	5.56	17.50	6.04	20.23	7.67	31.70	16.44
		0.50	8.77	2.06	8.81	2.10	8.96	2.15	9.24	2.28	10.06	2.73	13.53	4.64
		1.00	4.22	0.70	4.23	0.71	4.30	0.72	4.40	0.76	4.72	0.86	6.04	1.35
		2.00	2.12	0.32	2.13	0.34	2.17	0.37	2.23	0.42	2.44	0.50	3.06	0.47
		3.00	1.92	0.26	1.93	0.26	1.95	0.23	1.97	0.18	1.99	0.10	2.07	0.25
	0.95	0.00	7.23	1.83	7.36	1.88	7.67	2.03	8.13	2.21	9.86	3.01	19.59	8.75
		0.10	7.20	1.79	7.24	1.81	7.56	1.92	8.06	2.12	9.65	2.82	17.33	6.84
		0.25	5.70	1.26	5.72	1.26	5.88	1.32	6.06	1.40	6.67	1.60	8.90	2.57
		0.50	3.12	0.58	3.14	0.59	3.17	0.59	3.25	0.60	3.46	0.65	4.33	0.87
		1.00	1.94	0.23	1.95	0.23	1.96	0.20	1.97	0.17	2.00	0.11	2.22	0.41
		2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01	1.09	0.29
		3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
1.00	0.00	0.00	370.2	353.6	369.9	353.3	368.7	350.8	370.6	359.9	369.3	352.9	370.9	353.6
		0.10	106.2	87.81	107.8	89.03	109.1	90.98	113.8	95.36	125.1	106.5	165.3	148.1
		0.25	26.60	14.42	26.85	14.55	27.29	14.81	28.26	15.65	31.27	17.84	44.53	29.13
		0.50	10.60	3.63	10.63	3.61	10.81	3.76	11.15	3.94	12.14	4.43	16.30	6.89
		1.00	4.91	1.10	4.94	1.12	5.02	1.13	5.13	1.17	5.53	1.33	7.12	1.95
		2.00	2.55	0.52	2.56	0.52	2.64	0.51	2.69	0.51	2.84	0.51	3.49	0.66
		3.00	1.98	0.14	1.99	0.12	1.99	0.12	2.00	0.11	2.02	0.15	2.39	0.49
	0.25	0.00	372.1	353.7	371.9	354.9	369.2	356.3	370.3	356.2	371.3	355.9	368.7	352.9
		0.10	102.8	84.74	103.3	85.51	105.3	86.66	109.6	92.49	119.9	102.9	159.7	139.5
		0.25	25.40	13.50	25.56	13.44	26.13	13.93	27.23	15.01	29.70	16.59	42.53	27.58
		0.50	10.19	3.41	10.25	3.45	10.46	3.56	10.72	3.70	11.68	4.25	15.75	6.56
		1.00	4.77	1.07	4.78	1.05	4.85	1.08	4.97	1.12	5.34	1.25	6.86	1.85
		2.00	2.51	0.51	2.54	0.51	2.57	0.51	2.62	0.52	2.75	0.51	3.39	0.63
		3.00	1.97	0.16	1.98	0.17	1.98	0.14	1.99	0.13	2.01	0.12	2.30	0.46
	0.50	0.00	370.1	355.1	368.0	353.7	368.6	347.4	371.8	354.7	368.5	347.3	369.3	352.2
		0.10	86.39	69.58	87.83	69.51	89.60	71.53	92.92	74.39	101.5	83.09	139.9	122.6
		0.25	21.72	10.61	21.85	10.64	22.31	11.09	23.03	11.52	25.46	13.38	35.81	21.72
		0.50	8.94	2.78	8.98	2.79	9.15	2.88	9.41	2.99	10.23	3.41	13.61	5.37
		1.00	4.30	0.89	4.31	0.89	4.38	0.92	4.49	0.95	4.80	1.05	6.13	1.52
		2.00	2.25	0.39	2.27	0.40	2.28	0.42	2.33	0.45	2.48	0.51	3.08	0.53
		3.00	1.92	0.35	1.93	0.33	1.95	0.31	1.97	0.27	1.99	0.17	2.09	0.29
	0.95	0.00	368.6	358.2	369.1	353.5	368.8	349.9	371.8	358.7	367.3	351.9	368.1	354.0
		0.10	18.65	8.47	18.91	8.73	19.36	8.90	19.87	9.36	21.84	10.82	30.61	17.43
		0.25	6.21	1.57	6.25	1.62	6.34	1.65	6.51	1.69	7.00	1.90	9.16	2.91
		0.50	3.16	0.55	3.18	0.55	3.22	0.56	3.28	0.59	3.49	0.65	4.37	0.91
		1.00	1.96	0.21	1.96	0.20	1.97	0.18	1.98	0.15	2.00	0.10	2.27	0.42
		2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01	1.10	0.30
		3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
1.25	0.00	0.00	20.59	10.93	20.95	11.20	21.61	11.74	22.78	12.55	26.72	15.75	49.31	34.46
		0.10	19.68	10.22	19.99	10.55	20.53	10.90	21.68	11.66	25.46	14.63	44.67	30.98
		0.25	16.16	7.50	16.26	7.54	16.68	7.77	17.41	8.29	19.63	9.78	30.27	17.97

(*continued*)

Table 1. Continued.

			$\sigma_m^2 \div \sigma^2$											
			No error		0.1		0.2		0.3		0.5		1	
δ	ρ	γ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
		0.50	9.98	3.78	10.00	3.77	10.16	3.85	10.51	4.05	11.46	4.50	15.52	6.92
		1.00	4.91	1.34	4.94	1.38	5.02	1.38	5.13	1.44	5.53	1.58	7.12	2.19
		2.00	2.55	0.56	2.56	0.56	2.64	0.56	2.68	0.57	2.83	0.59	3.48	0.74
		3.00	1.96	0.22	1.97	0.22	1.98	0.21	1.99	0.20	2.02	0.22	2.39	0.50
	0.25	0.00	20.53	10.99	20.76	11.07	21.61	11.70	22.84	12.57	26.94	15.82	49.02	34.46
		0.10	19.68	10.22	19.89	10.20	20.54	10.73	21.60	11.54	25.08	14.10	44.34	30.34
		0.25	15.84	7.24	16.01	7.34	16.48	7.68	17.20	8.13	19.30	9.57	29.33	17.14
		0.50	9.67	3.58	9.74	3.67	9.90	3.73	10.19	3.88	11.12	4.35	14.96	6.60
		1.00	4.77	1.30	4.78	1.31	4.85	1.34	4.97	1.36	5.34	1.49	6.86	2.05
		2.00	2.51	0.54	2.54	0.54	2.57	0.55	2.60	0.56	2.74	0.58	3.39	0.70
		3.00	1.94	0.25	1.94	0.24	1.95	0.24	1.97	0.20	2.01	0.20	2.30	0.47
	0.50	0.00	19.88	10.48	20.04	10.53	20.74	11.05	22.10	12.13	25.80	14.90	47.38	33.62
		0.10	18.95	9.59	19.10	9.69	19.82	10.21	20.81	10.82	24.25	13.36	41.81	27.76
		0.25	14.71	6.47	14.84	6.64	15.28	6.85	15.83	7.20	17.76	8.36	26.70	14.92
		0.50	8.65	3.08	8.73	3.10	8.87	3.19	9.08	3.30	9.88	3.68	13.24	5.46
		1.00	4.29	1.10	4.31	1.10	4.37	1.12	4.48	1.16	4.79	1.24	6.12	1.71
		2.00	2.24	0.43	2.26	0.44	2.27	0.46	2.32	0.48	2.47	0.53	3.07	0.60
		3.00	1.80	0.40	1.81	0.39	1.84	0.37	1.88	0.33	1.94	0.24	2.08	0.33
	0.95	0.00	8.03	2.75	8.10	2.82	8.33	2.92	8.75	3.13	10.03	3.76	17.06	8.00
		0.10	7.68	2.56	7.74	2.56	7.95	2.69	8.31	2.86	9.43	3.34	14.91	6.26
		0.25	5.64	1.67	5.71	1.67	5.79	1.69	5.96	1.77	6.53	1.99	8.74	2.93
		0.50	3.15	0.78	3.18	0.79	3.21	0.79	3.28	0.79	3.48	0.85	4.36	1.07
		1.00	1.88	0.34	1.89	0.33	1.90	0.32	1.93	0.29	1.99	0.25	2.27	0.45
		2.00	1.00	0.01	1.00	0.01	1.00	0.01	1.00	0.02	1.00	0.04	1.14	0.35
		3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2.00	0.00	0.00	4.67	1.56	4.70	1.59	4.80	1.62	4.94	1.69	5.45	1.90	7.87	3.02
		0.10	4.66	1.55	4.70	1.57	4.80	1.62	4.93	1.67	5.43	1.88	7.84	2.96
		0.25	4.61	1.54	4.67	1.54	4.77	1.61	4.90	1.65	5.42	1.87	7.77	2.95
		0.50	4.49	1.46	4.52	1.47	4.60	1.50	4.74	1.54	5.22	1.76	7.35	2.63
		1.00	3.90	1.19	3.92	1.20	3.99	1.22	4.10	1.27	4.46	1.38	5.96	1.94
		2.00	2.55	0.70	2.56	0.71	2.64	0.72	2.67	0.73	2.83	0.77	3.48	0.96
		3.00	1.92	0.45	1.93	0.45	1.94	0.44	1.98	0.44	2.02	0.44	2.39	0.58
	0.25	0.00	4.66	1.58	4.70	1.58	4.78	1.62	4.94	1.67	5.43	1.87	7.87	3.01
		0.10	4.65	1.56	4.70	1.58	4.75	1.59	4.92	1.66	5.45	1.88	7.84	2.97
		0.25	4.61	1.55	4.65	1.53	4.75	1.58	4.89	1.64	5.38	1.85	7.69	2.90
		0.50	4.45	1.44	4.47	1.46	4.58	1.49	4.73	1.54	5.18	1.72	7.31	2.62
		1.00	3.85	1.18	3.86	1.18	3.94	1.20	4.06	1.25	4.39	1.34	5.86	1.90
		2.00	2.51	0.67	2.54	0.68	2.57	0.69	2.60	0.70	2.74	0.75	3.39	0.92
		3.00	1.87	0.45	1.88	0.44	1.90	0.44	1.93	0.44	2.02	0.43	2.39	0.55
	0.50	0.00	4.53	1.51	4.57	1.51	4.65	1.54	4.82	1.61	5.29	1.81	7.63	2.91
		0.10	4.52	1.49	4.56	1.50	4.65	1.55	4.80	1.59	5.28	1.81	7.62	2.84
		0.25	4.49	1.49	4.52	1.48	4.63	1.54	4.75	1.58	5.22	1.77	7.49	2.77
		0.50	4.30	1.38	4.32	1.39	4.39	1.41	4.55	1.46	4.99	1.63	7.04	2.47
		1.00	3.62	1.09	3.63	1.10	3.70	1.12	3.79	1.14	4.11	1.25	5.43	1.73
		2.00	2.24	0.60	2.26	0.60	2.27	0.61	2.32	0.62	2.47	0.67	3.07	0.82
		3.00	1.72	0.48	1.73	0.48	1.74	0.47	1.79	0.46	1.88	0.42	2.19	0.45
	0.95	0.00	2.26	0.61	2.27	0.61	2.30	0.62	2.37	0.64	2.58	0.70	3.53	0.98
		0.10	2.25	0.60	2.26	0.61	2.30	0.61	2.36	0.64	2.56	0.69	3.52	0.97
		0.25	2.21	0.60	2.23	0.61	2.27	0.61	2.32	0.63	2.51	0.69	3.44	0.95
		0.50	2.10	0.58	2.10	0.59	2.14	0.59	2.19	0.61	2.37	0.65	3.16	0.87
		1.00	1.70	0.56	1.70	0.56	1.73	0.55	1.77	0.56	1.88	0.57	2.27	0.63
		2.00	1.00	0.25	1.00	0.24	1.00	0.25	1.00	0.27	1.00	0.31	1.14	0.45
		3.00	1.00	0.01	1.00	0.02	1.00	0.02	1.00	0.02	1.00	0.03	1.00	0.05
3.00	0.00	0.00	2.58	0.85	2.59	0.86	2.64	0.87	2.70	0.88	2.92	0.94	3.86	1.25
		0.10	2.58	0.85	2.58	0.85	2.63	0.86	2.70	0.88	2.92	0.94	3.86	1.26
		0.25	2.58	0.85	2.58	0.84	2.63	0.86	2.69	0.88	2.91	0.94	3.86	1.25
		0.50	2.55	0.83	2.56	0.83	2.60	0.85	2.67	0.87	2.89	0.93	3.82	1.23

(*continued*)

Table 1. Continued.

			$\sigma_m^2 \div \sigma^2$											
			No error		0.1		0.2		0.3		0.5		1	
δ	ρ	γ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
		1.00	2.47	0.80	2.48	0.80	2.51	0.80	2.59	0.82	2.77	0.89	3.65	1.14
		2.00	2.17	0.68	2.17	0.67	2.21	0.68	2.25	0.69	2.40	0.73	3.06	0.90
		3.00	1.80	0.57	1.81	0.57	1.84	0.57	1.86	0.57	1.99	0.58	2.39	0.66
	0.25	0.00	2.57	0.84	2.58	0.84	2.64	0.87	2.69	0.88	2.91	0.94	3.85	1.26
		0.10	2.57	0.84	2.58	0.83	2.63	0.86	2.69	0.88	2.91	0.94	3.85	1.25
		0.25	2.57	0.84	2.58	0.85	2.62	0.86	2.69	0.88	2.91	0.95	3.85	1.26
		0.50	2.54	0.84	2.56	0.83	2.60	0.85	2.66	0.86	2.88	0.93	3.81	1.24
		1.00	2.45	0.79	2.47	0.79	2.51	0.80	2.57	0.82	2.77	0.87	3.64	1.16
		2.00	2.13	0.67	2.15	0.67	2..18	0.68	2.23	0.69	2.38	0.72	3.01	0.89
		3.00	1.76	0.56	1.77	0.57	1.79	0.57	1.83	0.57	1.95	0.58	2.38	0.65
	0.50	0.00	2.51	0.81	2.53	0.82	2.57	0.84	2.64	0.85	2.84	0.91	3.76	1.21
		0.10	2.52	0.81	2.53	0.82	2.57	0.83	2.63	0.85	2.84	0.90	3.74	1.20
		0.25	2.50	0.81	2.52	0.82	2.56	0.83	2.63	0.85	2.83	0.91	3.73	1.20
		0.50	2.48	0.79	2.49	0.80	2.54	0.82	2.60	0.83	2.80	0.89	3.70	1.18
		1.00	2.36	0.75	2.39	0.76	2.43	0.76	2.49	0.79	2.68	0.84	3.49	1.10
		2.00	2.02	0.64	2.03	0.64	2.07	0.65	2.11	0.65	2.24	0.68	2.82	0.83
		3.00	1.64	0.56	1.64	0.56	1.67	0.56	1.70	0.56	1.81	0.56	2.19	0.60
	0.95	0.00	1.31	0.47	1.32	0.48	1.35	0.49	1.37	0.50	1.49	0.53	1.93	0.56
		0.10	1.31	0.47	1.32	0.47	1.34	0.49	1.37	0.50	1.48	0.53	1.93	0.56
		0.25	1.31	0.48	1.32	0.48	1.34	0.49	1.36	0.50	1.47	0.53	1.92	0.55
		0.50	1.29	0.46	1.30	0.47	1.31	0.47	1.35	0.49	1.45	0.52	1.87	0.55
		1.00	1.23	0.43	1.23	0.43	1.25	0.44	1.27	0.45	1.36	0.50	1.74	0.56
		2.00	1.00	0.27	1.00	0.27	1.00	0.29	1.00	0.30	1.00	0.34	1.14	0.48
		3.00	1.00	0.10	1.00	0.11	1.00	0.12	1.00	0.13	1.00	0.14	1.00	0.24

0.25, 0.50, and 0.95, of correlation coefficient (ρ) between study and auxiliary variables are used to study the impact of their relationship on the chart efficiency. By using these ρ values, the correlation coefficient ρ^* between the variance of study variable $Var_{(y_j)}$ and variance of auxiliary variable $Var_{(w_j)}$ is calculated and get the values ρ^*= 0.0, 0.05, 0.23, and 0.89 respectively. It indicates that every increase in ρ result in increase in ρ^*, although increase in ρ^* is comparatively low. For in-control production process, we find the width of control limits i.e., L= 2.709 to get ARL_0=370 for this study assuming that no shift in mean $\gamma = 0$ and no shift in variance $\delta = 1$. In order to select samples to calculate *ARLs* and *SDRLs,* sample size has been determined as $n = 5$ and smoothing constant $\lambda = 0.05$.

Table 1 shows values of *ARLs* and *SDRLs* for different combinations to observe the effect of measurement error under columns showing values of $\sigma_m^2 \div \sigma^2$ from 0.1 to 1 using the covariate model with $A = 0$ and $B = 1$. The no error column shows performance without measurement error that these *ARLs* are calculated without any effect of measurement error i.e., $\sigma_m^2 \div \sigma^2 = 0.0$. Table 1 reveals that as correlation coefficient is increased from 0.0 to 0.95, the efficiency of the control chart is increased by decreasing *ARLs,* as decreasing *ARLs* mean to detect shift at an early stage. Use of auxiliary information in this control chart causes an increase in the efficiency of the chart for detection of minor shifts. As shifts in mean γ increase from 00 to 0.1, 0.25 and so on up to 3.00, the *ARLs* show a decreasing trend. A similar trend is observed for shifts in variance from unit value to decreasing as well as increasing values of shifts like 0.75, 0.50, 0.25, 1.25, 1.50 and so on up to 3.0, that every variance shift results in decreasing *ARLs*. Hence, it is clear from Table 1 that under all the columns *ARLs* are decreased with

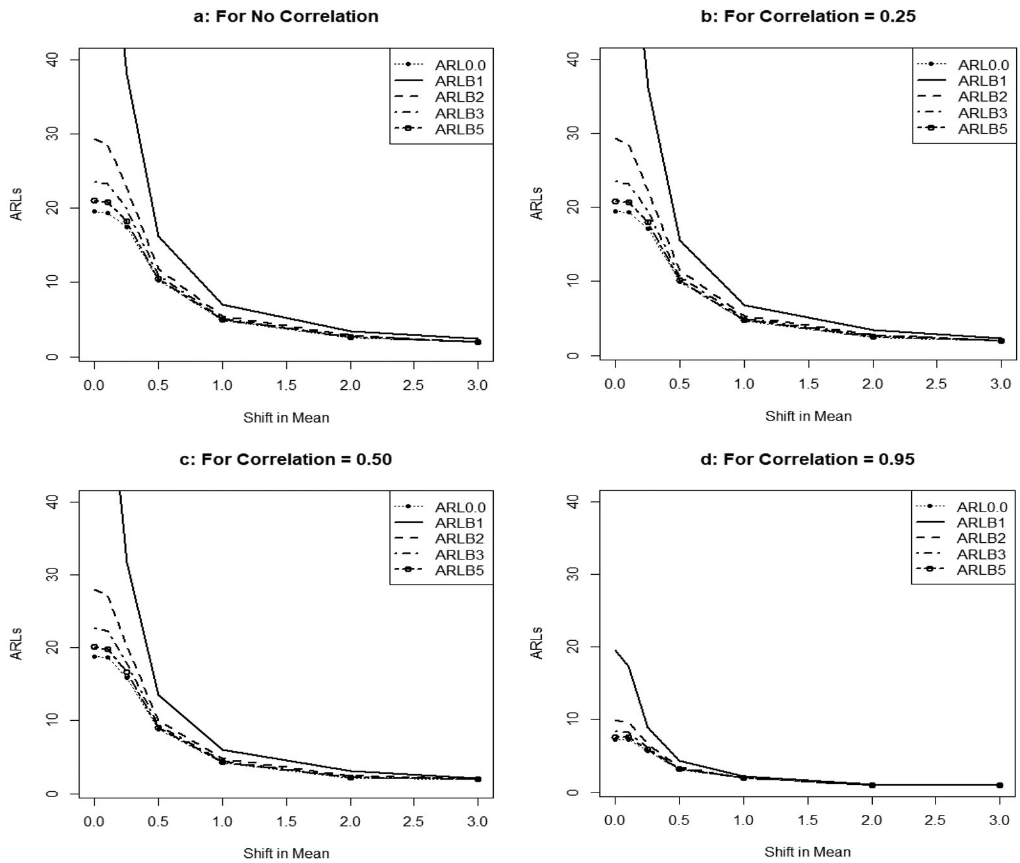

Figure 1. Max-EWMAMEAI control chart for $\delta = 0.75$, $\sigma_m^2 \div \sigma^2 = 1$ and different values of B.

every shift in mean or variance or combination of both, even minor shifts are detected through decrease in *ARLs*. From these trends, we can say that the use of auxiliary information increased the efficiency of our proposed control chart and this chart is eligible to detect process shifts jointly at the earliest. However, Table 1 also tells us that chart efficiency is affected by the measurement error.

In Table 1 different values of $\sigma_m^2 \div \sigma^2$ uncover that as the value of this ratio i.e., 0.1 is added in the calculations, it increased the values of *ARLs* as compared with *ARLs* under no error column. Increase in *ARLs* warn us that detection of process shifts has been delayed as compared with no error column, which has decreased the efficiency of the proposed control chart. When the error variance ratio $\sigma_m^2 \div \sigma^2$ is further increased from 0.1 to 0.2 and up to 1.0, the efficiency is further reduced for each single error value. It can be construed from this discussion that measurement error affected the efficiency of the Max-EWMAMEAI chart.

In addition to the ratio $\sigma_m^2 \div \sigma^2$, we also try to study the effect of the slope of the covariate model i.e., B, that how much the slope can reduce error effect to increase the efficiency of our proposed control chart. For this purpose, we prepare four graphs in Figure 1 rather to show the calculations just like Table 1. These graphs are based on *ARLs* for different values of B. Graphs are only for variance shift $\delta= 0.75$ which is 0.25

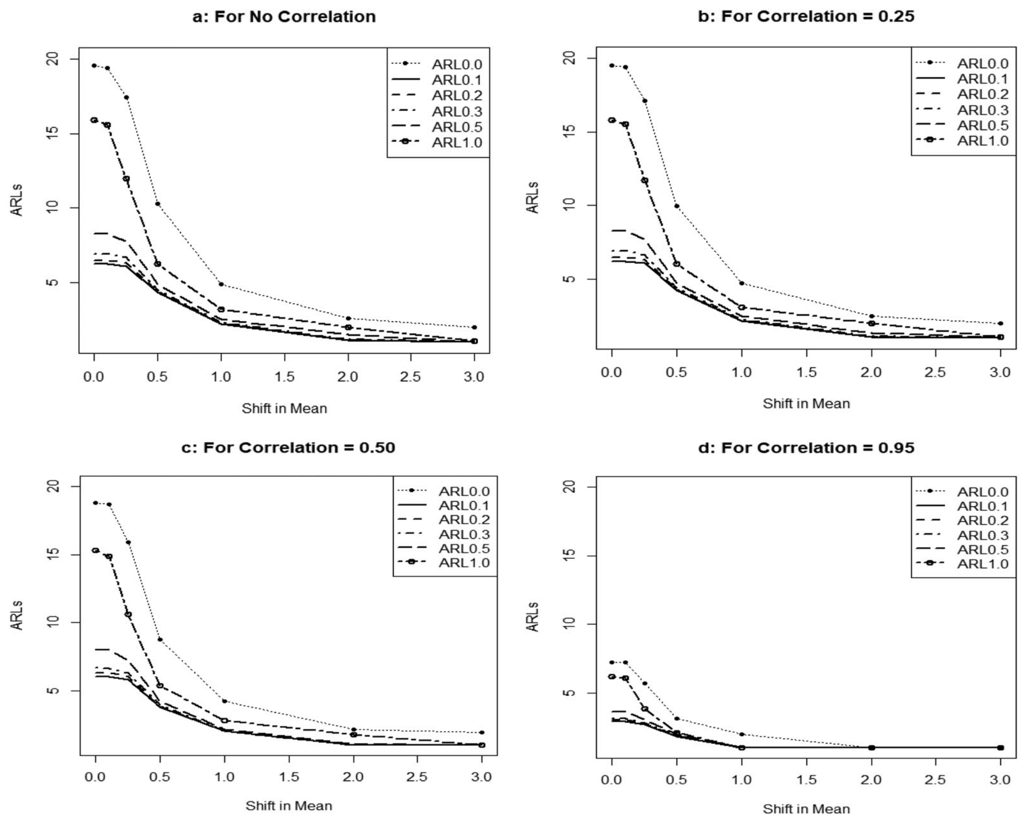

Figure 2. Max-EWMAMEAI chart for $\delta = 0.75,\ k = 5,\ B = 1$ and different values of $\sigma_m^2 \div \sigma^2$.

change from unit variance. All other variance shifts have the same trend from one shift to another, just like in Table 1. The graph line of $ARL0.0$ is for no measurement error where $B = 1$ and $\sigma_m^2 \div \sigma^2 = 0.0$. However, for other graph lines $\sigma_m^2 \div \sigma^2 = 1$ is fixed and B adopted different values for Figure 1.

Four graphs *a, b, c* and *d*, are presented in Figure 1 for four values of the correlation coefficient between quality characteristic and auxiliary variable. Figure 1 shows each graph has four graph lines for four different values of slope B used in the covariate model, in addition to a line graph of $ARL0.0$, whereas line graph of $ARLB1$ is for $B = 1$, $ARLB2$ is for $B = 2$, $ARLB3$ is for $B = 3$ and $ARLB5$ is for $B = 5$. It is clearly shown in four graphs with fixed $\sigma_m^2 \div \sigma^2 = 1$, that line of no error graph $ARL0.0$ is below the other four lines which means that measurement error has affected the efficiency of the chart negatively and delayed the detection of process shifts in mean and variance for joint monitoring. However, for fixed measurement error i.e., $\sigma_m^2 \div \sigma^2 = 1$, every increase in B from 1 to 2, 3, and 5, causes a decrease in ARL values. Line of $ARLB1$ is above all lines which indicates that it has the maximum negative effect due to $\sigma_m^2 \div \sigma^2 = 1$ and causes a delay in shift detection as compared to no error line. As the values of slope B are increased, their lines for each increase are below the previous one i.e., line for $B2$ is below than $B1$, $B3$ is below $B2$ and $B5$ is below $B3$, which highlight the effect of each increase in slope of covariate model that every increasing value of B

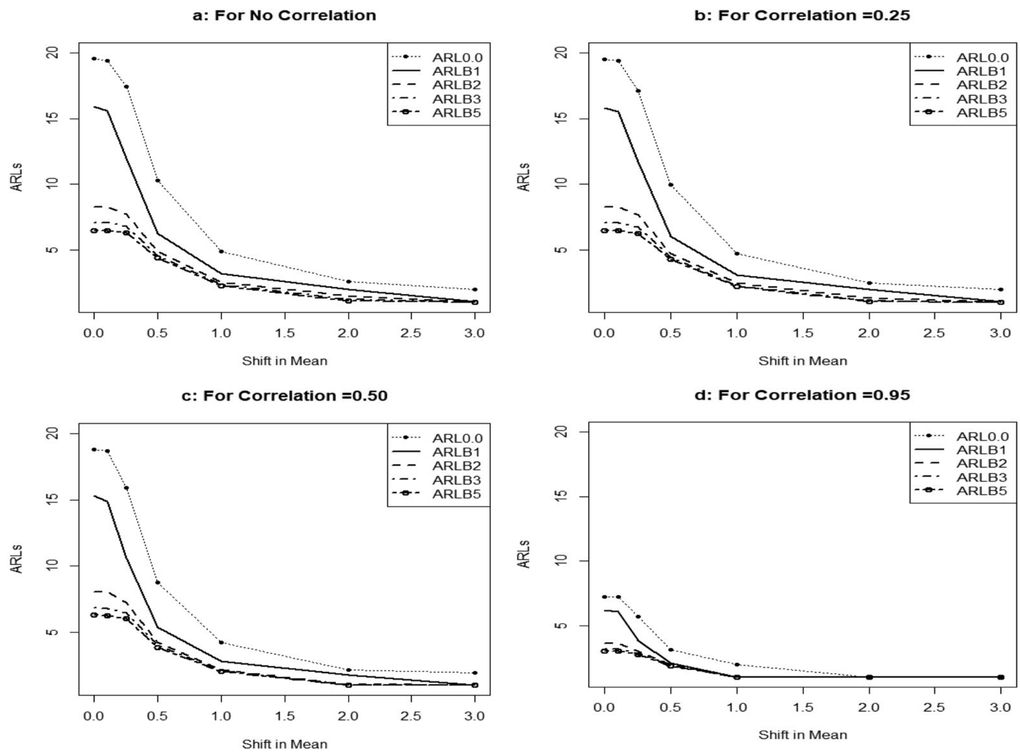

Figure 3. Max-EWMAMEAI chart for $\delta = 0.75,\ k = 5,\ \sigma_m^2 \div \sigma^2 = 1$ and different values of B.

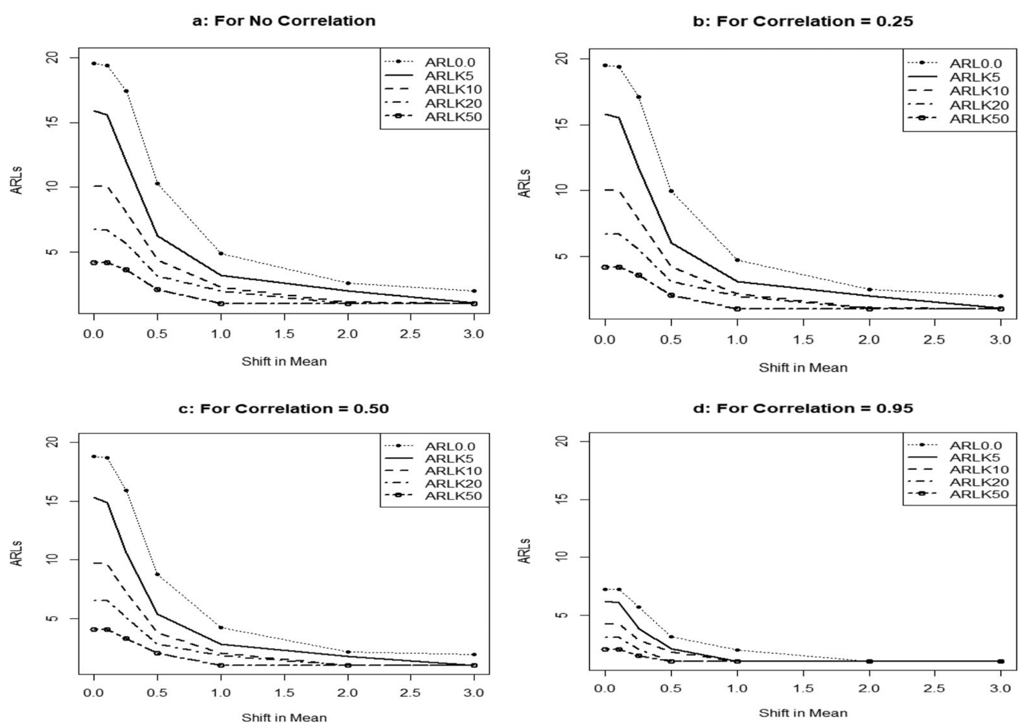

Figure 4. Max-EWMAMEAI chart for $\delta = 0.75,\ \sigma_m^2 \div \sigma^2 = 1,\ B = 1$ and different values of k.

increases the efficiency of chart by early detection of shifts through smaller *ARLs*. We can say that every increase in *B* reduces the error effect. It is also clear from these graphs just like Table 1 that every increase in correlation coefficient increases the chart efficiency by decreasing *ARL* lines which are very much lower for 0.95 correlation in graph *d*. To reduce the effect of measurement error, multiple measurements of each observation or sample can play an important role for which Figure 2 is prepared.

Figure 2 also shows graph lines of *ARLs*, where the effect of correlation coefficient can be seen just like Figure 1. However, five measurements i.e., $k = 5$, have been taken for each sample using covariate model, where the error variance ratio $\sigma_m^2 \div \sigma^2$ has its effect to delay the indication of process shift with every increase from 0.1 to unit. In addition to graph line of *ARL*0.0, there are five more lines *ARL*0.1, *ARL*0.2, *ARL*0.3, *ARL*0.5 and *ARL*1.0 for values of error variance ratio $\sigma_m^2 \div \sigma^2 = 0.1$, 0.2, 0.3, 0.5 and 1.0 respectively. It is observed that the line of no error graph is above all the other lines in contrast to Figure 1, and values of no error column are smaller than all other columns in Table 1. It shows that multiple measurements have reduced the *ARLs* even for each increase in the error variance ratio and most of the lines are touching $ARL = 1$ at or after mean shift of 2.0. Hence, we can say that multiple measurements reduce the error effect and make the *ARL* lines even lower than no error line. Therefore, we further investigate the impact of multiple measurements along with the effect of slope *B* in the covariate model, which is shown in Figure 3.

Figure 3 depicts that *ARLs* have been reduced even below 20 just like Figure 2, as compared with Figure 1 where *ARLs* are even greater than 40 particularly for $B = 1$. This is the impact of multiple measurements using the covariate model that process shifts are indicated at the earliest for joint monitoring. The relevancy of graph lines is the same as in Figure 1. In Figure 1 the line graph of *ARL*0.0 for no error was below all the other lines, which is now above all the other lines in Figure 3. We can say that multiple measurements have too much impact to reduce the error effect using the covariate model. The no error *ARLs* show delay in detecting the process shifts while multiple measurements have reduced the impact of error so much that detection of process shifts has become very early even unit or near to unit for the mean shift of 2 and above, for joint monitoring of mean and variance. The graph lines of $B = 1$, 2, 3, and 5 are below the *ARL*0.0 which are above the *ARL*0.0 in Figure 1. However, *B* has the same impact to reduce the error effect with every increase in its value just like in Figure 1. After analyzing the impact of $k = 5$ measurements of each selected unit or sample in Figures 2 and 3, we are encouraged to study the impact of multiple measurements by increasing values of *k* even more than 5.

Figure 4 is prepared by plotting the *ARLs* considering $\sigma_m^2 \div \sigma^2 = 1$, $B = 1$ and different values of *k* just like Maravelakis, Panaretos, and Psarakis (2004). The no error line is the same as is in previous charts. However, *ARLK*5 is graph line for $k = 5$, *ARLK*10 for $k = 10$, *ARLK*20 for $k = 20$ and *ARLK*50 is for $k = 50$. It is clear from the line graphs of four charts in Figure 4 that as *k* is increased from 5 to 50, the *ARL* graphs are reduced sufficiently with each increase in *k*. For $k = 50$ the all line graphs are below lines for $k = 5$ and have become closer to unit values of *ARLs* at mean shift 1 and onward. Figure 4 *d* i.e., for correlation = 0.95 shows that the graph line touches unit value even at the mean shift of 0.5.

Given the above discussion, we can say that our proposed chart is an efficient control chart for joint monitoring to detect the small shifts in mean and variance which shows the power of this chart. The efficiency of this chart to detect process mean and variance shifts for joint monitoring is badly affected by the measurement error. However, measurement error can be reduced by taking multiple measurements of each observation or sample which of course, requires extra time and cost.

Algorithm for proposed Max-EWMAMEAI chart is briefly presented as:

(i) First of all, fix the value of desired in-control ARL_0 at $\gamma = 0$ and $\delta = 1$.
(ii) Fix parameters and constants $A = 0,\ B = 1,$ and $\lambda = 0.05$.
(iii) For in-control process considering desired ARL_0, determine the value of L.
(iv) Select sample of size $n = 5$ and calculate $Max - EWMAMEAI$ statistic and UCL.
(v) If the calculated statistic remains below the UCL, the process is in-control for this sample, and select another sample.
(vi) However, if $Max - EWMAMEAI$ statistic is beyond UCL, it shows that some shift has occurred in the production process and this out of control sample number is the run length (RL). For out of control situation stop the process and investigate the cause of the shift.
(vii) Then select another sample to complete 50,000 iterations.

3.2. Unknown parameters case

We have studied the proposed control chart in Sec. 3.1 when population parameters for the distribution of quality characteristics i.e., mean and variance of study variable are known and the results are shown in Table 1 and Figures 1–4. Let us analyze the situation when the parameters of study variable Y are unknown and are estimated in phase-I. Ghosh, Reynolds, and Yer (1981), Chakraborti (2000), Schoonhoven, Riaz, and Does (2009), Shabbir and Awan (2016), and Noor-Ul-Amin, Khan, and Sanaullah (2019) have discussed the cases with unknown parameters. We also discuss the cases when population parameters of study variable are not known in advance and have to be estimated from the reference sample in phase-I as follows:

(i) Considering in-control process, prepare a mean control chart having known μ_y using 1000 observations.
(ii) For in-control process, generate a reference sample of 1000 observations having a bivariate normal distribution with known parameters.
(iii) Plot these 1000 reference sample points on the mean control chart and observe whether the sample points are in control. If all points are within the control limits of the mean chart, then proceed further for estimation of unknown parameters.
(iv) From in-control 1000 sample points, estimate population parameters of study variable Y in three combinations / cases as:
   (a) $\sigma_y^2,\ \rho_{yw}$ and β_{yw} are unknown,
   (b) σ_y^2 and ρ_{yw} are unknown but β_{yw} is known,
   (c) σ_y^2 and β_{yw} are unknown but ρ_{yw} is known.

Table 2. *ARLs* of Max-EWMAMEAI chart with different values of $\sigma_m^2 \div \sigma^2$ for case (a).

			$\sigma_m^2 \div \sigma^2$		
δ	γ	ρ_{yw}	0.1	0.3	0.7
1.00	0.00	0.25	367.2	371.9	368.3
		0.50	366.8	370.4	368.5
		0.75	369.9	365.8	367.6
		0.95	364.0	363.9	362.1

Then calculate the ARL_0 values through Mote Carlo simulations for our proposed chart using known and estimated values of (a), (b) and (c) as in Tables 2–4. Other values are the same as used in Table 1.

We also tried other than 1000 sample points to estimate the population parameters as described above. It was observed that with every increase in number of sample points, the estimates become closer to the parameters of bivariate normal distribution. Smaller estimates from small observations provide small ARL values which may lead to misleading decision. Therefore, choice of 1000 reference sample points is considered suitable for unknown parameters case.

3.3. Linearly increasing variance

Let us examine the error effect using linearly increasing variance approach. It is assumed that population parameters are known while error variance is not constant but changes linearly with change in variable X. In this case, all other things remain the same as explained earlier but the error term $\varepsilon \sim N(0,(C+D\mu))$, where $\sigma_m = C + D\mu$. For this situation, the changes in statistics are given while the rest of the procedures and statistics will remain the same as in case of constant error variance with $\varepsilon \sim N(0,\sigma_m)$.

For linearly increasing variance using covariate model, we can write the transformed estimator for mean as:

$$U_{je} = \frac{U_j - E(U_j)}{Var(U_j)} = \frac{\bar{Y} - (A + B\mu)}{\sqrt{\{(B^2\sigma_p^2 + C + D\mu)(1-\rho^2)\}/n}}, \tag{11}$$

which follows the standard normal distribution as $U_{je} \sim N(0,1)$, whereas, $\frac{(n-1)S_{w_j}^2}{\sigma_w^2} \sim \chi^2_{(n-1)}$, $\frac{(n-1)S_{y_j}^2}{B^2\sigma_p^2+C+D\mu} \sim \chi^2_{(n-1)}$, $Var_{(w_j)} = \phi^{-1}\left[H\left\{\frac{(n-1)S_{w_j}^2}{\sigma_w^2},(n-1)\right\}\right] \sim N(0,1)$, and $Var_{(y_j)} = \phi^{-1}\left[H\left\{\frac{(n-1)S_{y_j}^2}{B^2\sigma_p^2+C+D\mu},(n-1)\right\}\right] \sim N(0,1)$ for in-control production process.

Table 5 has been prepared for linearly increasing variance having $B=1$, $C=0$ and different values of D while Table 6 is constructed for $B=1$, $D=1$ and different values of C, using $\lambda = 0.05$ as used in Table 1 and Figures 1–4. Both the tables are prepared for no change in variance i.e., $\delta = 1$, the correlation coefficient between study and auxiliary variables i.e., $\rho = 0.5$ and different mean shift combinations, having $L=2.709$ for in-control process $ARL_0 = 370$.

Table 5 reveals that there is no shift in variance and no change in correlation coefficient, as both the changes have already been studied in Table 1, mean shifts from zero to 3, no error column is the same as in Table 1 just to make comparison with similar

Table 3. *ARLs* of Max-EWMAMEAI chart with different values of $\sigma_m^2 \div \sigma^2$ for case (b).

			$\sigma_m^2 \div \sigma^2$		
δ	γ	ρ_{yw}	0.1	0.3	0.7
1.00	0.00	0.25	369.8	372.4	371.9
		0.50	368.2	366.3	371.8
		0.75	367.1	372.6	366.9
		0.95	370.6	362.2	367.4

pattern, but impact of different values of D from 1 to 5. Table 5 tells us that for no shift in mean and variance, the ARL_0 are 370 for any value of D, while shift in mean has the same positive effect on the chart efficiency as discussed in Table 1. However, when the value of D increases from 1 to 2, 3, and 5, the *ARLs* are increased respectively as compared with no error column, which indicates that every increase in *ARL* decreases the chart efficiency due to increasing error component in the covariate model, as D is multiplied with μ.

Table 6 shows the impact of C component of linearly increasing variance, which increases from zero to 1, 2, and 3. As the value of C increases, the values of *ARLs* become larger with every increase but this enlargement of *ARLs* is comparatively lesser than that of Table 5 which is larger due to impact of D. It is expected as the C is a simple addition for linearly increasing error variance while D is multiplied with μ.

4. Real life example

To support the simulation results, we choose a real life example which was also used by Sanusi et al. (2017) and Noor-Ul-Amin, Khan, and Sanaullah (2019). Data set is chosen from nonisothermal continuous stirred tank chemical reactor (CSTR). The temperature from outside is taken as a study variable Y and cooling water temperature is considered as an auxiliary variable W for this example. Data comprising 1024 values are collected on sample basis after every minute. Data from in-control points are used to calculate parameters.

The estimates of mean and standard deviation from sample values are $\bar{Y} = 368.23$, $\bar{W} = 365.02$ and $S_y = 0.4671,\ S_w = 0.5439$ respectively. Correlation coefficient is $\rho_{yw} = 0.71$ between Y and W. These estimates are considered as parameters and used to generate values assuming both the variables follow the bivariate normal distribution. For in-control process first we arrived $ARL_0 = 370$ with $L = 2.709$ without any process shift considering $\gamma = 0,\ \delta = 1,\ \lambda = 0.05,\ \rho^* = 0.47$ and $\mu_0 = 368.23$. Then we calculate *ARLs* and *SDRLs* for variance shift i.e., $\delta = 0.75$, shifts in mean i.e., $\gamma = 0.0$, 0.10, 0.25, 0.50, 1.00, 2.00, 3.00 and for different levels of error variance i.e., $\sigma_m^2 \div \sigma^2 = 0.1$, 0.2, 0.3, 0.5, 1 in order to see the effect of measurement error using covariate model. Same combinations of values are used to study the effect of multiple measurements i.e., $k = 5$. Figure 5 depicts *ARLs* calculated for real life data selected from continuous stirred tank chemical reactor (CSTR).

Figure 5a tells that no error line i.e., *ARL*0.0 is at the minimum level of 17 for no mean shift but is below 17 for every increase in the mean shift. Then measurement error is introduced for 0.1 and *ARL*0.1 is little above *ARL*0.0. For increase in error from 0.1 to 0.2, the *ARL*0.2 is higher than *ARL*0.1, similarly every increase in error

Table 4. *ARLs* of Max-EWMAMEAI chart with different values of $\sigma_m^2 \div \sigma^2$ for case (c).

			$\sigma_m^2 \div \sigma^2$		
δ	γ	ρ_{yw}	0.1	0.3	0.7
1.00	0.00	0.25	370.4	368.3	369.8
		0.50	373.8	366.6	374.2
		0.75	369.4	363.4	369.1
		0.95	377.7	374.4	367.7

Table 5. *ARLs* and *SDRLs* of Max-EWMAMEAI chart for linearly increasing variance with different values of *D*.

			D									
			No error		1		2		3		5	
δ	ρ	γ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	0.50	0.00	370.2	353.6	370.4	356.7	370.1	354.2	369.3	351.1	369.4	350.3
		0.10	106.2	87.81	282.1	265.6	318.8	303.3	330.4	314.7	347.4	332.8
		0.25	26.60	14.42	129.8	111.1	185.5	166.3	222.7	205.8	266.1	250.7
		0.50	10.60	3.63	46.23	30.42	75.90	58.41	100.5	81.99	141.7	123.9
		1.00	4.91	1.10	16.69	7.26	26.19	13.91	34.79	20.78	51.24	35.27
		2.00	2.55	0.52	7.25	2.02	10.50	3.58	13.34	5.22	18.45	8.36
		3.00	1.98	0.14	4.70	1.03	6.62	1.76	8.21	2.46	11.01	3.88

Table 6. *ARLs* and *SDRLs* of Max-EWMAMEAI chart for linearly increasing variance with different values of *C*.

			C									
			No error		0		1		2		3	
δ	ρ	γ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	0.50	0.00	370.2	353.6	370.4	356.7	369.2	351.7	369.7	354.2	370.4	353.6
		0.10	106.2	87.81	282.2	265.6	291.5	274.9	298.4	280.2	297.5	277.9
		0.25	26.60	14.42	129.8	111.1	137.1	117.9	143.9	126.0	150.9	134.7
		0.50	10.60	3.63	46.23	30.42	49.29	33.86	52.30	36.57	55.50	39.13
		1.00	4.91	1.10	16.69	7.26	17.71	7.87	18.78	8.55	19.68	9.12
		2.00	2.55	0.52	7.25	2.02	7.61	2.18	7.94	2.33	8.27	2.46
		3.00	1.98	0.14	4.70	1.03	4.92	1.11	5.12	1.18	5.33	1.26

makes the *ARL* graph line higher than previous. Ultimately *ARL*1 is so high that it touches 250. It is clear from these trends that every increase in error causes an increase in *ARLs* which decreases the efficiency and delays process shift detection. To reduce the measurement error effect, we apply the strategy of multiple measurements i.e., $k = 5$ and shows results in Figure 5b. It is clearly shown that multiple measurements of the same sample reduce the effect of measurement error and increase the efficiency of the proposed chart by early detection of process shift for all error values except *ARL*1 which is above *ARL*0.0. However, *ARL*1 is reduced from 250 to 70 due to multiple measurements. Overall measurement error effect is reduced sufficiently and increased the chart efficiency by taking multiple measurements of the same sample.

For the implementation of our proposed control chart using parameters of this real life example, we generated 25 samples of size n = 5. First 15 samples for in-control process with $ARL_0 = 370$ and $L = 2.709$ with no process shift, while last 10 samples with mean shift $\gamma = 0.5$ and calculated plotting statistics i.e., Max-EWMAMEAI as well as UCL for each sample. Then we enhanced the mean shift from 0.5 to 1.0 and calculated

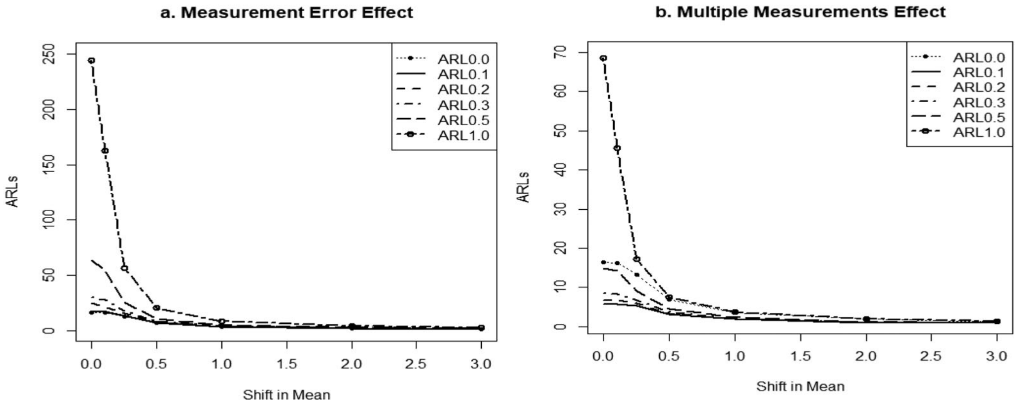

Figure 5. *ARLs* from continuous stirred tank chemical reactor.

plotting statistics for these last 10 samples. Afterward introduced measurement error i.e., $\sigma_m^2 \div \sigma^2 = 0.3$ and 0.7, in the last 10 samples and calculated plotting statistics for both levels of error. The plotting statistics and UCL are depicted in Figure 6.

Figure 6 is the actual implementation of our proposed control chart. UCL is the upper control limit showing the first 15 samples below the UCL being in-control process. However, the last 10 samples are showing process shifts as well as error effect on the chart. The Mx1 chart is for the mean shift of 0.5 which goes out of control (OOC) at 21st sample, while Mx2 chart is for the mean shift of 1.0 which pulls the process OOC at an early stage at 18th sample, showing that process shift is detected at an early stage due to larger process shift in this control chart. With same process shift of 1.0, the ME0.3 chart is for measurement error 0.3 which goes OOC at 20th sample i.e., later than Mx2, while ME0.7 chart of measurement error 0.7 further delays the process shift due to increased error and goes OOC at 23rd sample. We can say that measurement error causes a delay in process shift detection for real life data used in this example.

5. Main findings

From the discussion on tables and figures, the main findings can be highlighted as:

(i) Max-EWMAMEAI chart is eligible to detect small shifts for joint monitoring of mean and variance in a production process.

(ii) Every shift in mean and / or variance increases the efficiency of the proposed control chart by decreasing *ARLs*.

(iii) The use of auxiliary information enhances the chart efficiency and *ARL* values are rapidly reduced from $ARL_0 = 370$ with every process shift.

(iv) Every increase in the correlation between quality characteristic and auxiliary variable causes a decrease in the values of *ARLs* which indicates that the efficiency of this chart is increased with every increase in the correlation coefficient. It is found that as the use of auxiliary information is enhanced by increase in correlation, the chart efficiency is also increased.

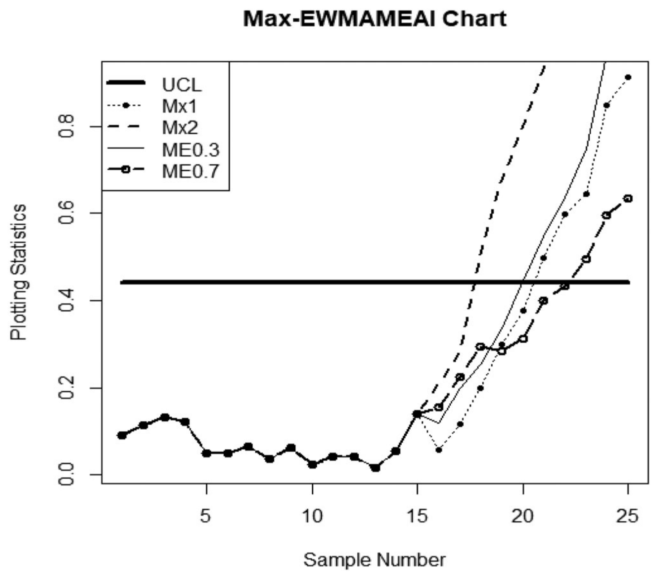

Figure 6. Max-EWMAMEAI control chart showing process shift and measurement error.

(v) Measurement error in the covariate model, affects the efficiency of this chart by increasing *ARLs* for every increase in the error as compared with no error values of *ARLs*. The auxiliary information based efficient chart is also affected by the measurement error.

(vi) Every increase in the value of the slope *B* of the covariate model from 2 to 3 and 5, decreases *ARLs* which reduces the error effect. If we take multiple measurements of the same sample, every increase in *B* enhances the efficiency of this chart for joint monitoring, even *ARLs* become smaller than no error case, which is proved according to the result of Linna and Woodall (2001).

(vii) Linearly increasing variance also decreases the chart efficiency with every increase in variance which is proved according to Maravelakis, Panaretos, and Psarakis (2004).

(viii) Multiple measurements technique reduces the effect of measurement error by using the covariate model, through reduction in *ARLs* showing overall early detection of shifts.

(ix) Multiple measurements reduce *ARLs* so much that most of the *ARLs* approach unit value which means that multiple measurements enable the proposed chart to detect process shifts at the earliest.

6. Conclusion

Statistical process control provides the technique of control charts which is applicable for maintenance of the quality of the industrial products. Many control charts have been developed to monitor the process shifts in mean and variance individually as well as jointly. Use of auxiliary information in the control charts has increased their efficiency for joint monitoring. However, error in measurement of units is affecting the efficiency of control charts. We developed Max-EWMA control chart with measurement error using auxiliary information and name it Max-EWMAMEAI control chart for joint monitoring of mean and variance shifts in the production process. From the results shown in tables and figures, it can be concluded that the proposed chart is affected by measurement error using covariate model. To reduce the error effect, multiple measurements using the covariate model are applied and results are proved effective.

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