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Microscopic theory of the activated behavior of the quantized Hall effect

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ABSTRACT

The thermally activated behavior of the gate defined narrow Hall bars is studied by analyzing the existence of the incompressible strips within a Hartree-type approximation. We perform self-consistent calculations considering the linear response regime, supported by a local conductivity model. We investigate the variation of the activation energy depending on the width of samples in the range of $2d \sim [1-10]\,\mu$m. We show that the largest activation energy of high-mobility narrow samples, is at the low field edge of Hall filling factor 2 plateau (exceeding half of the cyclotron energy), whereas for relatively wide samples the higher activation energy is obtained at the high field edge of Hall plateau. In contrast to the single-particle theories based on the localization of electronic states, we found that the activation energy is almost independent of the properties of the density of states.

1. Introduction

A two-dimensional electron system (2DES) subjected to strong magnetic fields perpendicular to the 2DES exhibits the integer quantum Hall effect (IQHE) [1]. Due to the extremely high reproducibility of the certain quantized resistance values, QHE stands as a resistance standard [2]. The appearance of narrow resistivity peaks separated by deep minima is a defining feature of the QHE [3]. The standard explanation of the IQHE is based on single-particle picture which accepts the Landau quantization *and* localization of electronic states as key points [4]. The Coulomb interaction between the electrons is neglected in this picture and so it is unable to explain several important features, such as high reproducibility and precise quantization [5,6]. Recently, a microscopic interpretation of the IQHE that incorporates the electron–electron interaction explicitly [7,8], provides a prescription to calculate the Hall and longitudinal resistances (R_{xy} and R_{xx}, respectively) explicitly under experimental conditions. It is stated that a quantitative theory that describes the activated behavior has to include an analysis of the longitudinal and Hall conductivity and their dependencies on the temperature, magnetic field, current density, sample widths and the other material properties [9]. There have been several attempts to explain the physics behind the key features of activation of the quantum Hall effect. For different temperature regimes various transport processes have been proposed [10,11]. At intermediate temperatures ($10\,\mathrm{K} < T < 20\,\mathrm{K}$) conductance is predominantly determined by thermal activation of electrons. The temperature dependence of the conductivity σ_{xx} is thermally activated with an activation energy E_a corresponding to the energy difference between the Fermi energy and the mobility edge [12]. In our calculation scheme 'mobility edge' corresponds to disappearance of the bulk incompressible region, which overlaps with the bulk picture in the asymptotic limit of infinite sample size and highly disordered system. In the single particle theories, it is expected that the largest activation energy is obtained if the Fermi energy resides at the midpoint between two Landau levels [4], i.e. at the center of the B field interval where R_{xx} vanishes. However, in the literature strong deviations are reported when considering high mobility narrow samples [6,13].

2. Theoretical model and method

The purpose of the present paper is to summarize the results on activated behavior of high-mobility Hall bars studied within a microscopic theory which avoids any localization assumptions. A gate defined narrow Hall bar system with sample width $2d$ is constructed by in-plane metallic side gates kept in zero potential, i.e. $V(-d) = V(d) = 0$ dictating the boundary conditions to the solution of the relevant Poisson equation. The 2DES is depleted from the edges by an amount of $|b|/d$ which resides in the $z = 0$ plane. The spatial distribution of the electron number density in x-direction is denoted by $n_{\mathrm{el}}(x)$ and translational invariance in y-direction is assumed. Electron system is populated by ionized Si-donors with average density n_0 which are distributed homogeneously in the same plane with the 2DES. The confinement potential $V_{\mathrm{bg}}(x)$ determined by these background charges is

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obtained from the solution of the Poisson equation yielding the kernel

$$K(x,x') = \ln\left|\frac{\sqrt{(d^2-x^2)(d^2-x'^2)}+d^2-xx'}{(x-x')d}\right|, \quad (1)$$

via

$$V_{bg}(x) = \frac{2e^2}{\overline{\kappa}}\int_{-d}^{d} dx'\, K(x,x')n_0, \quad (2)$$

which leads to

$$V_{bg}(x) = -E_0\sqrt{1-(x/d)^2}, \quad E_0 = 2\pi e^2 n_0 d/\overline{\kappa}. \quad (3)$$

In above equations $\overline{\kappa}$ is an average background dielectric constant of the material. The Hartree potential due to the Coulomb interaction between the electrons is obtained by using the same electrostatic kernel as

$$V_H(x) = \frac{2e^2}{\overline{\kappa}}\int_{-d}^{d} dx'\, K(x,x')n_{el}(x'). \quad (4)$$

Hence the electrons are subjected to an effective potential defined as

$$V(x) = V_{bg}(x) + V_H(x). \quad (5)$$

We neglect the antisymmetry condition for the Fermionic wave functions in calculation of electron density, considering spinless particles. Such simplification is justified due to the small g^*-factor (≈ -0.44) of a 2DES induced on a GaAs/AlGaAs, the Zeeman energy is much smaller than the Landau energy. Since the confining potential for electrons varies smoothly over the extend of the eigenfunction we are allowed to employ the self-consistent Thomas–Fermi approximation which describes realistically the electronic distribution as,

$$n_{el}(x) = \int dE\, D(E)f(E), \quad (6)$$

where $D(E)$ is the density of states described by self-consistent Born approximation [14] in the presence of strong magnetic field and $f(E)$ is the Fermi–Dirac distribution function which is position dependent via the local electrostatic potential [5]. After solving Eqs. (5) and (6) self-consistently, the current distribution is obtained by utilizing the local version of the conventional transport theory, i.e. Ohm's law, which takes into account implicitly the peculiar screening effects in 2DES under the high magnetic fields, however, avoids any localization assumptions. The nonlocal effects on the conductivities, adopted from the results of self-consistent Born approximation, are simulated by coarse-graining the conductivity tensor according to

$$\hat{\overline{\sigma}} = \frac{1}{2\lambda}\int_{-\lambda}^{\lambda} d\chi\, \hat{\sigma}(x+\chi), \quad (7)$$

where $\lambda = \lambda_F/2$ and λ_F is the mean particle distance, i.e. Fermi wavelength. Since without any disorder it is not possible to define the conductivity, we considered impurity potentials described by a Gaussian

$$\upsilon(\mathbf{r}) = \frac{V_0}{\pi R^2}\exp\left(-\frac{r^2}{R^2}\right) \quad (8)$$

with the single particle range R of the order of the spacing between 2DES and doping layer, where V_0 is the impurity strength with relevant dimensions. A configuration average of such impurity potentials lead to the spectral function $A_N(E)$ defined as

$$A_N(E) = \frac{2}{\pi\Gamma_N}\sqrt{1-\frac{E-E_N}{\Gamma_N}}. \quad (9)$$

The temperature dependencies of the longitudinal and the Hall resistances are obtained from the position dependent two-dimensional resistivity tensor [14] and are studied at magnetic field intervals within the integer Hall plateau with filling factor $\nu = 2$ considering high mobility samples.

3. Results and discussion

In this work we consider the extremely clean GaAs/(AlGa)As heterostructures where local DOS is almost a delta function and no long-range potential fluctuations exist. We fixed the background charges to the density $n_0 = 4\times10^{11}\,\mathrm{cm}^{-2}$ and obtained the corresponding density profile at zero temperature and zero field potential. Subsequent step is finding iteratively the positions of incompressible strips (ISs) for the finite temperature and finite field, where we fixed the depletion length to $|b|/d = 0.9$. Throughout the work we used the Fermi energy E_F^0 corresponding to the electron density at the center of the sample ($n_{el}(0)$) as a reference energy.

The numerical results in Fig. 1 depicts R_{xy} and R_{xx} as a function of magnetic field. Whenever the system is in the plateau regime we conclude that an IS of filling factor $\nu = 2$ is formed *somewhere* across the Hall bar, since backscattering is absent.

Within this strip the electrostatic potential varies by the amount of a cyclotron energy and current flows only in this IS, whereas the adjacent regions are compressible strips where nearly perfect screening occurs and partially filled Landau level is pinned to the Fermi energy. Once the ISs become leaky, i.e. if the strip widths become narrower than the averaging length, scattering between $\nu > 2$ and $\nu < 2$ compressible states is

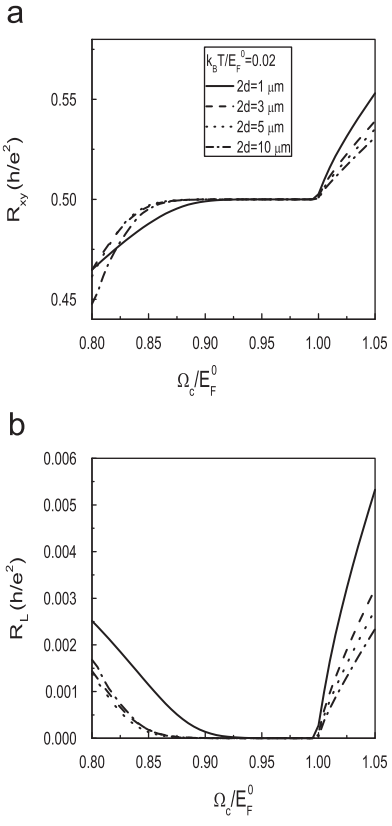

Fig. 1. Calculated Hall (a) and (b) longitudinal resistances versus scaled magnetic field $\hbar\omega_c/E_F^0$, with $\omega_c = eB/m^*c$ for different values of sample widths. The parameters are $R = 20$ nm, $\Gamma/\hbar\omega_c = \Gamma/\Omega_c = 0.05$, and $k_BT/E_F^0 = 0.02$.

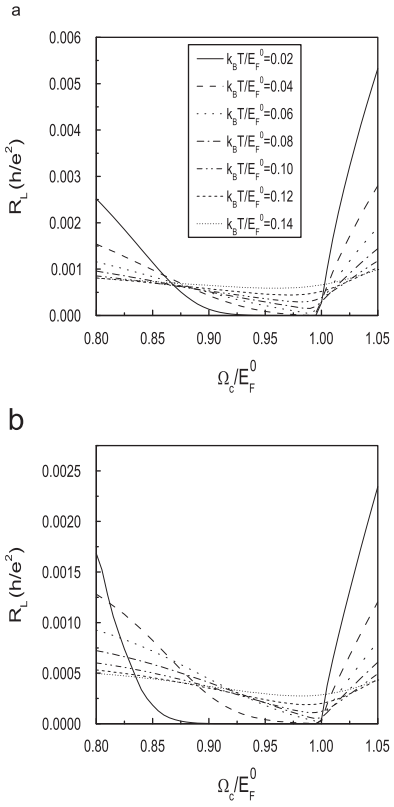

Fig. 2. Dependency of the calculated R_{xx} as a function of magnetic field for different temperatures.

possible, then the quantized Hall effect disappears. Standard Hall wafers contain a 2DES of typical density $n_{el}(0) \leq 4 \times 10^{11}$ cm^{-2}, hence we find a finite magnetic field interval in which IS with integer value of the local filling factor 2 exist. For the B values in this interval, the deviation of the Hall resistance from the quantized values increases with increasing temperature. To calculate the activation energy it is crucial to identify the exact B value for $\overline{\nu}=2$, where bar stands for the average filling factor. For standard Hall bars, where edge effects are suppressed by the disorder effects and the activation energy is calculated at the center of the plateau, i.e $\overline{\nu}=2$. In contrast, for narrow samples where edge effects are predominant and electron distribution is no longer flat. Since the behavior of the crossing of classical curve and the low temperature curve is asymmetric with respect to the center of the plateau, it is not straightforward to locate the B value corresponding to $\overline{\nu}=2$. In order to define the B value where the activation energy should be calculated, we present the temperature dependence of the longitudinal resistance for narrow and wide samples in Fig. 2.

The relevant B value is determined where the longitudinal resistance remains minimal while increasing the temperature. For the $2d=1\,\mu$m this critical B value is found to be at $B_a=\Omega_c/E_F^0=0.990$ while for $2d=10\,\mu$m at $B_a=\Omega_c/E_F^0=0.995$.

In order to clarify the relation between the formation of the ISs and the global resistances, we present the calculated local filling factor profile as a function of the scaled lateral coordinate and varying magnetic field in Fig. 3, as gray scale. At sufficiently large B field $\Omega_c/E_F^0>1$, the local filling factor $\nu(x)$ is less than 2 everywhere and system is completely compressible. Reducing the B leads to the formation of IS at the center of the sample with $\nu(x)=2$ while gradually decreases to zero toward the edge of the sample. Further decrease of B, enforces the ISs with $\nu=2$ move toward the sample edge with narrowing its width, meanwhile filling factor of the central region increases.

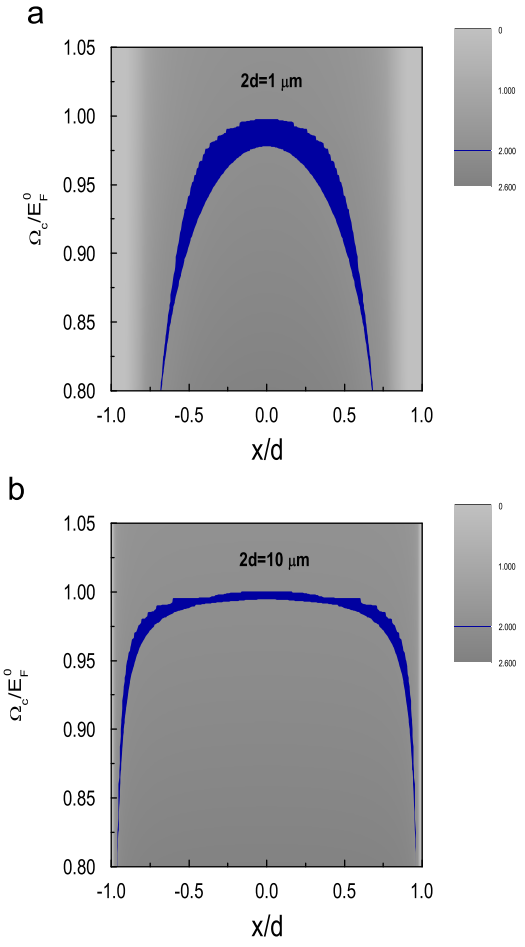

Fig. 3. Gray scale plot of the averaged filling factor profile versus position x and magnetic field Ω_c/E_F^0 at $k_BT/E_F^0=0.02$. The regions of IS with $\nu=2$ are indicated.

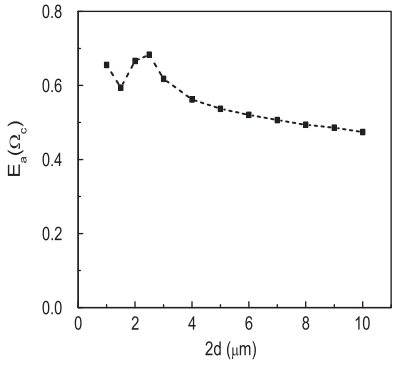

Fig. 4. Dependence of the activation energy of the $\nu=2$ Hall plateau on the width of the samples.

As a next, at low bias currents, thermally activated resistance is investigated by the Arrhenius plot at the relevant magnetic field values. The activation energies were extracted from fitting

$$R_{xx}(T,B_a)=R_{xx}^0\exp(-E_a/2k_BT)$$

to the maximum slopes of the data points. Results for E_a are shown in Fig. 4. For relatively wide samples, activation energy is

calculated at the high field edge of Hall plateau whereas for the narrower samples activation energy is obtained at the low field edge. Energies for narrower samples exceeds half of the cyclotron energy of the interval between Landau levels. However, with increase in width of sample the asymptotic decrease in activation energy recovers the well known values. The nonmonotonic behavior observed at narrower samples $2d < 2\,\mu m$ is closely related with the formation of the large bulk incompressible region. Such anomalies are discussed elsewhere [15].

In summary, the temperature dependence of the longitudinal conductivity has been studied in the high-mobility gate defined narrow Hall bar samples with well developed IQHE plateaux in magnetic field interval corresponding to filling factor $\nu = 2$. Activation energies obtained by fitting the data to the Arrhenius law are calculated at the magnetic field where the longitudinal resistance remains the smallest with increase in temperature.

In contrast to the single-particle theories, we found that the activation energy depends strongly on the width of sample. The highest values of activation for narrow samples are obtained at the low field edge of Hall plateau whereas for wider samples this values shifts to the high field edge. Activation energies, for extremely narrow samples exceeding the half of the cyclotron energy, decreases asymptotically with increase of sample width. We suggest that the enhanced contribution to the activation energy is connected with the width of ISs which promotes the thermally activated conduction.

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References

[1] K.v. Klitzing, G. Dorda, M. Pepper, Phys. Rev. Lett. 45 (1980) 494.
[2] M.E. Cage, B.F. Field, R.F. Dziuba, S.M. Girvin, A.C. Gossard, D.C. Tsui, Phys. Rev. B 30 (1984) 2286.
[3] K.v. Klitzing, Philos. Trans. R. Soc. A 363 (2005) 2203.
[4] P. Svoboda, G. Nachtwei, C. Breitlow, S. Heide, M. Cukr, Semicond. Sci. Technol. 12 (1997) 264.
[5] A. Siddiki, R.R. Gerhardts, Phys. Rev. B 70 (2004) 195335.
[6] J. Horas, A. Siddiki, J. Moser, W. Wegscheider, S. Ludwig, Physica E 40 (2008) 1130.
[7] A. Siddiki, R.R. Gerhardts, Int. J. Mod. Phys. B 21 (2007) 1362.
[8] A. Siddiki, Europhys. Lett. 87 (2009) 17008.
[9] J. Matthews, M.E. Cage, J. Res. Natl. Inst. Stand. Technol. 110 (2005) 497.
[10] R.M. Rubinger, G.M. Ribeiro, A.G. de Oliviera, H.A. Albuquerque, R.L. da Silva, C.P.L. Rubinger, et al., Semicond. Sci. Technol. 21 (2006) 1681.
[11] M. Furlan, Phys. Rev. B 57 (1998) 14818.
[12] M. Furlan, Physica B 249–251 (1998) 123.
[13] H.Z. Zheng, K.K. Choi, D.C. Tsui, G. Weimann, Phys. Rev. Lett. 55 (1985) 1144.
[14] T. Ando, A.B. Fowler, F. Stern, Rev. Mod. Phys. 54 (1982) 437.
[15] S. Sakiroglu, A. Siddiki, G. Oylumluoglu, U. Erkarslan, I. Sokmen, Unpublished, 2009.