# A Method for Decision Making Problems by Using Graph Representation of Soft Set Relations 

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#### Abstract

Soft set theory, which was defined by D. Molodtsov, has a rich potential for applications in several fields of life. One of the successful application of the soft set theory is to construct new methods for Decision Making problems. In this study, we are introducing a method using graph representation of soft set relations to solve Decision Making problems. We have successfully applied this method to various examples.


KEY WORDS: Decision Making Problem, Representation of Soft Set Relations, Soft Sets.

## 1 INTRODUCTION

THERE are real life problems in engineering, social and medical sciences, economics etc. involving imprecise data that can be solved by mathematical principles based on uncertainty and imprecision. Most of the times, traditional methods are limited due to their uncertainties. Though theory of probability, fuzzy set theory, intuitionistic fuzzy sets, vague sets, theory of interval mathematics, rough set theory etc. may be utilized as efficient tools to deal with diverse types of uncertainties and imprecision embedded in a system, they have their inherit difficulties as pointed out by Molodtsov (1999).

Molodtsov (1999) introduced soft set theory and successfully applied it in several directions such as the smoothness of the functions, game theory, operation research, Riemann integration, Perron integration, probability, theory of measurement. Maji, Biswas and Roy (2003) studied equality of two soft sets, soft subset and soft super set of a soft set, complement of a soft set, null soft set, absolute soft set and soft binary operations. Some soft and fuzzy soft algebraic structures e.g. soft groups, soft rings, fuzzy soft moduls were also studied by many researchers (Acar, et. al. 2010; Aktaş and Çağman, 2007; Gunduz and Bayramov, 2011a, 2011b; Ozturk, Gunduz and Bayramov, 2013). The soft set relation was introduced by Babitha and Sunil (2010). In another study Babitha and Sunil (2011) transitive closures and orderings on soft sets were introduced. Later in 2012, some properties related to soft set relations were extended by Park, Kim and Kwun (2012).

Moreover, Çağman and Enginoğlu (2010) defined soft matrices, which are representation of soft sets. Out of its several advantages, one is to store and manipulate matrices, hence the soft sets can be entered in a computer.

After these advances, the soft set theory became an important mathematical tool for vagueness. Zhang (2014) introduced the Interval soft set, which is a combination of interval set and soft set, and applied the interval soft set to construct a decision making algorithm. In addition, the concept of soft intervals, whose special case is interval soft set, was defined by Tanay and Yaylalı (2015) Constructing new Decision Making Algorithms is very important for social life problems involving imprecise data. (Ballı and Turker, 2017; Wang and Wang, 2016; Zeinalova, 2014) are some recent studies on decision making methods that are based on multi-criteria decision making and decision making under Z-information.

Representing soft structures in an innovative and effective way is very important to improve the soft set theory. Herein, we have developed a new tool to visualize soft set relation using directed graphs and successfully applied it to solve decision making problems given in previous studies (Tanay and Yaylal, 2015; Zhang, 2014). We have used some literature (Grimaldi, 2004) elementary definitions in the graph theory such as graph, subgraph etc.

## 2 PRELIMINARIES

LET's recall some basic notions in the soft set theory:

Definition 2.1. (Molodtsov, 1999) Let $U$ be an initial universal set and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$ and $A \subseteq E$. A pair ( $F, A$ ) is called a soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow P(U)$.
Definition 2.2. (Maji, et. al. 2003) For two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$, we say that $(F, A)$ is soft subset of $(G, B)$ if:
i) $A \subset B$, and
ii) $\forall e \in A, F(e)$ and $G(e)$ are identical approximations.
it is denoted by $(F, A) \widetilde{\subset}(G, B)$.
Definition 2.3. (Maji, et. al. 2003) Let ( $F, A$ ) and $(G, B)$ be soft sets over a common universe $U$. The intersection of $(F, A)$ and $(G, B)$ is defined as the soft set ( $H, C$ ) satisfying the following conditions:
i) $C=A \cap B$.
ii) For all $x \in C, H(x)=F(x)$ or $G(x)$
(while two soft sets are the same).
in this case, we can write $(F, A) \widetilde{\cap}(G, B)=(H, C)$.
Definition 2.4. (Maji, et. al. 2003) Let ( $F, A$ ) and $(G, B)$ be soft sets over a common universe $U$. The union of $(F, A)$ and $(G, B)$ is defined as the soft set ( $H, C$ ) satisfying the following conditions:
i) $C=A \cup B$.
ii) For all $x \in C$,
$H(x)=\left\{\begin{array}{ccc}F(x) & \text { if } & x \in A-B, \\ G(x) & \text { if } & x \in B-A, \\ F(x) \cup G(x) & \text { if } & x \in A \cap B .\end{array}\right.$
in that case, we write $(F, A) \widetilde{\cup}(G, B)=(H, C)$.
Definition 2.5. (Babitha and Sunil, 2010) Let ( $F, A$ ) and $(G, B)$ be soft sets over a common universe $U$, then the Cartesian product of $(F, A)$ and $(G, B)$ is defined as $(F, A) \times(G, B)=(H, A \times B)$, where $H: A \times B \rightarrow P(U \times U)$ and $H(a, b)=F(a) \times G(b)$, where $(a, b) \in A \times B$.

The Cartesian product of three or more nonempty soft sets can be defined by generalizing the definition of the Cartesian product of two soft sets.
Definition 2.6. (Babitha and Sunil, 2010) Let ( $F$, $A$ ) and $(G, B)$ be soft sets over a common universe $U$, then a soft set relation from $(F, A)$ to $(G, B)$ is a soft subset of $(F, A) \times(G, B)$. In an equivalent way, the soft set relation $R$ on a soft set ( $F, A$ ) can be defined as follows in the parameterized form:
if $(F, A)=\{F(a), F(b), \ldots\}$, then $F(a) R F(b)$ iff $F(a) \times F(b) \in R$.

Definition 2.7. (Yang and Guo, 2011) Let ( $F, A$ ) be a soft set over $U$ and $R, Q$ be a soft set relations on $(F, A)$, then:

1) The complement of the soft set relation $R$ on a soft set $(F, A)$, denoted as $R^{C}$, is defined by $R^{C}=\{F(a) \times F(b): F(a) \times F(b) \notin R, a, b \in A\}$.
2) The inverse of the soft set relation $R$ on ( $F, A$ ), denoted as $R^{-1}$, is defined by $R^{-1}=\{F(b) \times F(a): F(a) \times F(b) \in R\}$.
3) The union of two soft set relation $R$ and $Q$ on $(F, A)$, denoted as on $R \cup Q$, is defined by $R \cup Q=\{F(a) \times F(b): F(a) \times F(b) \in R$ or $F(a) \times F(b) \in Q\}$.
4) The intersection of two soft set relation $R$ and $Q$ on $(F, A)$, denoted as on $R \cap Q$, is defined by $R \cap Q=\{F(a) \times F(b): F(a) \times F(b) \in R$ and $F(a) \times F(b) \in Q\}$.
Definition 2.8. (Yang and Guo, 2011) Let $R, Q$ be two soft set relations on $(F, A) . \quad \forall a, b \in A$, if $F(a) \times F(b) \in R \Rightarrow F(a) \times F(b) \in Q$, then we say that $R \subset Q$.
Definition 2.9. (Babitha and Sunil, 2010) Let $R$ be a soft set relation on $(F, A)$, then
5) $R$ is reflexive if $F(a) \times F(a) \in R, \forall a \in A$.
6) $R$ is symmetric if $F(a) \times F(b) \in R \Rightarrow F(b) \times F(a) \in R, \forall a, b \in A$.
7) $R$ is transitive if $F(a) \times F(b) \in R \quad$ and $F(b) \times F(c) \in R \Rightarrow F(a) \times F(c) \in R \quad$ for every $a, b, c \in A$.
Definition 2.10. (Babitha and Sunil, 2011)Let $R$ be a soft set relation on $(F, A)$, then $R$ is anti-symmetric if $F(a) \times F(b) \in R$ and $F(b) \times F(a) \in R$ imply that $F(a)=F(b), \quad \forall a, b \in A$.
Definition 2.11. (Yang and Guo, 2011) Let $I$ be a soft set relation on $(F, A)$. If for all $a, b \in A$ and $a \neq b, \quad F(a) \times F(a) \in I$, but $F(a) \times F(b) \notin I$, then $I$ is called as the identity soft set relation.
Definition 2.12. (Babitha and Sunil, 2010) Let ( $F, A$ ), $(G, B)$ and ( $H, C$ ) be three soft sets. Let $R$ be a soft set relation from $(F, A)$ to $(G, B)$ and $S$ be a soft set relation from $(G, B)$ to $(H, C)$. Then, a new soft set relation, the composition of $R$ and $S$ expressed as $S \circ R$ from $(F, A)$ to $(H, C)$, is defined as follows:
if $F(a)$ is in $(F, A)$ and $H(c)$ is in $(H, C)$, then $F(a) S \circ R H(c)$ iff there is some $G(b)$ in $(G, B)$ such that $F(a) R G(b)$ and $G(b) S H(c)$.

We use the notation $R^{n}$ for the $n^{\text {th }}$ composition of the relation $R$.

## 3 REPRESENTING A SOFT SET RELATION USING A DIRECTED GRAPH

THIS section states about some basic definitions in the graph theory, following the book named Discrete and Combinatorial Mathematics (Grimaldi, 2004).
Definition 3.1. (Grimaldi, 2004) A directed graph $G=(V, E)$, or digraph, consists of a set $V$ of vertices (or nodes) together with a set $E$ of edges (or arcs). The vertex $a \in V$ is called the initial vertex of the edge $(a, b) \in E$, while the vertex $b \in V$ is called the terminal vertex of this edge. The edge $(a, a)$ is called a loop. When a graph $G=(V, E)$ contains no loop, it is called loop-free. There is a path starting at $a \in V$ and ending at $b \in V(a \neq b)$. Such a path consists of a finite sequence of directed edges.
Definition 3.2. (Grimaldi, 2004) If $G=(V, E)$ is a graph, then $G_{1}=\left(V_{1}, E_{1}\right)$ is called a subgraph of $G$ if $\varnothing \neq V_{1} \subseteq V$ and $\varnothing \neq E_{1} \subseteq E$, where each edge in $E_{1}$ is incident with vertices in $V_{1}$.
Definition 3.3. (Grimaldi, 2004) Let $V$ be a set of $n$ vertices. The complete graph on $V$, denoted by $K_{n}$, is a loop free undirected graph, where for all $a, b \in V, a \neq b$, there is an edge $(a, b)$.
Definition 3.4. (Grimaldi, 2004) Let $G$ be a loop-free undirected graph on $n$ vertices. The complement of $G$, denoted $\bar{G}$, is the subgraph on $K_{n}$ consisting of the $n$ vertices in $G$ and all edges that are not in $G$.
Example 3.5. Let $(F, A)$ be a soft set over $U$ where $U=\left\{h_{1}, h_{2}, h_{3}, h_{4}, h_{5}\right\}, \quad A=\left\{e_{1}, e_{2}, e_{3}, e_{4}\right\} \quad$ and $F\left(e_{1}\right)=\left\{h_{1}, h_{2}, h_{5}\right\}, F\left(e_{2}\right)=\left\{h_{3}, h_{4}\right\}$, $F\left(e_{3}\right)=\left\{h_{2}, h_{4}\right\}, F\left(e_{4}\right)=\left\{h_{1}, h_{4}\right\}$.

Let the soft set relation $R$ on ( $F, A$ ) is given as: $R=\left\{F\left(e_{1}\right) \times F\left(e_{1}\right), F\left(e_{1}\right) \times F\left(e_{2}\right), F\left(e_{2}\right) \times F\left(e_{2}\right)\right.$, $F\left(e_{2}\right) \times F\left(e_{3}\right), F\left(e_{3}\right) \times F\left(e_{3}\right), F\left(e_{4}\right) \times F\left(e_{4}\right)$, We $\left.F\left(e_{4}\right) \times F\left(e_{1}\right)\right\}$.
can represent $R$ with a directed graph, named $G_{R}$, as follows: if we consider the set of vertices $V$ and set of edges $E$ as:
$V=\left\{F\left(e_{1}\right), F\left(e_{2}\right), F\left(e_{3}\right), F\left(e_{4}\right)\right\}$ and

$$
\begin{aligned}
& E=\left\{\left(F\left(e_{1}\right), F\left(e_{1}\right)\right),\left(F\left(e_{1}\right), F\left(e_{2}\right)\right),\right. \\
& \left(F\left(e_{2}\right), F\left(e_{2}\right)\right),\left(F\left(e_{2}\right), F\left(e_{3}\right)\right),\left(F\left(e_{3}\right),\right. \\
& \left.\left.F\left(e_{3}\right)\right),\left(F\left(e_{4}\right), F\left(e_{4}\right)\right),\left(F\left(e_{4}\right), F\left(e_{1}\right)\right)\right\}, \\
& \text { respectively }
\end{aligned}
$$

then the graph $G_{R}=(V, E)$ will be the Graph Representation of $R$.
An edge of the form $\left(F\left(e_{i}\right), F\left(e_{i}\right)\right)$ is illustrated by arc from the vertex $F\left(e_{i}\right)$ back to itself. Such an edge is called a loop.

The directed graph $G_{R}$ for the soft set relation $R$ can be illustrated as below:


Graph 1. Graph representation of soft set relation $R$.

### 3.1 A method for decision making problems using the graph representation of soft set relation

Zhang defined interval soft sets and used it to solve a decision making problem (Zhang, 2014). In his paper, interval soft sets are represented by a table from which, he derived an interval choice value. Based on his idea, we resolved same decision making problems as previously reported (Tanay and Yaylalı, 2015; Zhang, 2014) using the graph representation of soft set relations. Our method is discussed below with examples:
Example 3.6. A soft set ( $F, E$ ) describes the attractiveness of the houses that Mr . X is going to buy. Let $U$ be the set of houses under consideration, $U=\left\{h_{1}, h_{2}, h_{3}, h_{4}, h_{5}, h_{6}\right\}$ be the universal set: $E=\left\{e_{1}=\right.$ expensive, $e_{2}=$ beautiful, $e_{3}=$ wooden,
$e_{4}=$ cheap, $e_{5}=$ in green surrounding $\}$
be the parameter set. Let's define a soft set ( $F, E$ ) such that:
$F\left(e_{1}\right)=\left\{h_{2}, h_{3}\right\}, F\left(e_{2}\right)=\left\{h_{2}, h_{3}, h_{5}\right\}$,
$F\left(e_{3}\right)=\left\{h_{1}, h_{4}\right\}, F\left(e_{4}\right)=\left\{h_{1}\right\}, F\left(e_{5}\right)=\left\{h_{1}, h_{2}, h_{6}\right\}$.
Let Mr. X priorities are ranked as beautiful, in green surrounding, cheap, expensive and wooden houses. According to this priority ranking, we can define a soft set relation on $(F, E)$ as follows:

$$
\begin{aligned}
& R=\left\{F\left(e_{3}\right) \times F\left(e_{1}\right), F\left(e_{3}\right) \times F\left(e_{4}\right), F\left(e_{3}\right) \times F\left(e_{5}\right),\right. \\
& F\left(e_{3}\right) \times F\left(e_{2}\right), F\left(e_{1}\right) \times F\left(e_{4}\right), F\left(e_{1}\right) \times F\left(e_{5}\right), \\
& F\left(e_{1}\right) \times F\left(e_{2}\right), F\left(e_{4}\right) \times F\left(e_{5}\right), F\left(e_{4}\right) \times F\left(e_{2}\right), \\
& \left.F\left(e_{5}\right) \times F\left(e_{2}\right)\right\}
\end{aligned}
$$

And the graph representation of $R$ is as follows:


Graph 2. Graph representation of soft set relation $R$.

Step 1: Count the incoming and outgoing paths to each vertex and represent them with ordered pairs where first component is the number of incoming ways, while second component is the number of outgoing ways.

For the node $F\left(e_{1}\right)$ in Example 3.6, we have 1 incoming and 3 outgoing ways. Thus, we get the pair $(1,3)$ for the node $F\left(e_{1}\right)$. In this example, all ways are listed below:

$$
\begin{array}{ll}
F\left(e_{1}\right) & (1,3) \\
F\left(e_{2}\right) & (4,0) \\
F\left(e_{3}\right) & (0,4) \\
F\left(e_{4}\right) & (2,2) \\
F\left(e_{5}\right) & (3,1)
\end{array}
$$

Step 2: Examine the images of each $F\left(e_{i}\right)$ to determine the existence of the elements of universal set. For this purpose, we will see the images $F\left(e_{i}\right)$ as vectors and each of its element represents the existence of elements of universal set in same order. After applying the method to Example 3.6, the existence vectors are as follows:

| $(0,1,1,0,0,0)$ | for $F\left(e_{1}\right)=\left\{h_{2}, h_{3}\right\}$ |
| :--- | :--- |
| $(0,1,1,0,1,0)$ | for $F\left(e_{2}\right)=\left\{h_{2}, h_{3}, h_{5}\right\}$ |
| $(1,0,0,1,0,0)$ | for $F\left(e_{3}\right)=\left\{h_{1}, h_{4}\right\}$ |
| $(1,0,0,0,0,0)$ | for $F\left(e_{4}\right)=\left\{h_{1}\right\}$ |
| $(1,1,0,0,0,1)$ | for $F\left(e_{5}\right)=\left\{h_{1}, h_{2}, h_{6}\right\}$ |

Step 3: Find the incoming and outgoing values for each $F\left(e_{i}\right)$. One can find these values by adding the numbers of incoming and outgoing ways of $F\left(e_{i}\right)$ to each component of vectors. Incoming and outgoing values are evaluated below for Example 3.6. First
vectors represent incoming values while the second represent outgoing values for $h_{1}, h_{2}, h_{3}, h_{4}, h_{5}$ and $h_{6}$ in $F\left(e_{1}\right), \quad F\left(e_{2}\right), \quad F\left(e_{3}\right), \quad F\left(e_{4}\right)$, and $\quad F\left(e_{5}\right)$ respectively:

$$
\begin{array}{lcc}
F\left(e_{1}\right) & (0,2,2,0,0,0) & (0,4,4,0,0,0) \\
F\left(e_{2}\right) & (0,5,5,0,5,0) & (0,1,1,0,1,0) \\
F\left(e_{3}\right) & (1,0,0,1,0,0) & (5,0,0,5,0,0) \\
F\left(e_{4}\right) & (3,0,0,0,0,0) & (3,0,0,0,0,0) \\
F\left(e_{5}\right) & (4,4,0,0,0,4) & (2,2,0,0,0,2)
\end{array}
$$

Step 4: Compute the total incoming and outgoing values for each $h_{i}$. The Maximum incoming value provides us the choice. If there are more than one objects that have maximal incoming value, then the maximum outgoing value of the objects that have maximal incoming values will give us the choice.

Total incoming and outgoing values for $h_{1}, h_{2}, h_{3}, h_{4}, h_{5} \quad$ and $\quad h_{6} \quad$ are $(8,10),(11,7),(7,5),(1,5),(5,1)$ and $(4,2)$, respectively. Hence the house will be $h_{2}$ according to our algorithm.

The previous results that we have obtained in the stated steps can be summarized in Table 1, where vertices of graph are given in the first column while the elements of universal set are in the first row. Entries in the table show the existence, the change and the total values for each $h_{i}$ in the sets $F\left(e_{i}\right)$. Left subscript of $F\left(e_{i}\right)$ symbolizes the number of incoming ways while the right subscript symbolizes the number of outgoing ways. We can also use both subscripts of the entries to represent the effects of incoming and outgoing ways to each element of universal set which were found in Step 3.

Table 1. Summarized Algorithm for Example 3.6.

|  | $h_{1}$ | $h_{2}$ | $h_{3}$ | $h_{4}$ | $h_{5}$ | $h_{6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{1} F\left(e_{1}\right)_{3}$ | ${ }_{0} 0_{0}$ | ${ }_{2} 1_{4}$ | ${ }_{2} 1_{4}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ |
| ${ }_{4} F\left(e_{2}\right)_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{5} 1_{1}$ | ${ }_{5} 1_{1}$ | ${ }_{0} 0_{0}$ | ${ }_{5} 1_{1}$ | ${ }_{0} 0_{0}$ |
| ${ }_{0} F\left(e_{3}\right)_{4}$ | ${ }_{5} 1_{1}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{5} 1_{1}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ |
| ${ }_{2} F\left(e_{4}\right)_{2}$ | ${ }_{3} 1_{3}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ |
| ${ }_{3} F\left(e_{5}\right)_{1}$ | ${ }_{4} 1_{2}$ | ${ }_{4} 1_{2}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{4} 1_{2}$ |
|  | $(8,10)$ | $(11,7)$ | $(7,5)$ | $(1,5)$ | $(5,1)$ | $(4,2)$ |

Example 3.7. Let $U$ be the set of cars under consideration, $U=\left\{c_{1}, c_{2}, c_{3}, c_{4}, c_{5}, c_{6}, c_{7}\right\}$ and let $E$ be the parameter set such that:
$E=\left\{e_{1}=\right.$ diesel, $e_{2}=$ gasoline, $e_{3}=$ light color,
$e_{4}=$ dark color, $e_{5}=$ manuel, $e_{7}=$ new,
$e_{8}=$ second hand $\}$.

Let a soft set $(F, A)$ describes the attractiveness of the cars that Mr . X is going to buy. Consider $A=\left\{e_{1}=\right.$ diesel, $e_{2}=$ gasoline, $e_{3}=$ light color,
$\mathrm{e}_{4}=$ dark color, $\mathrm{e}_{5}=$ manuel, $\mathrm{e}_{7}=$ new, $\mathrm{e}_{8}=$ second hand $\}$ and
$F\left(e_{1}\right)=\left\{c_{1}, c_{3}, c_{5}\right\}, F\left(e_{2}\right)=\left\{c_{2}, c_{4}, c_{6}, c_{7}\right\}$,
$F\left(e_{3}\right)=\left\{c_{1}, c_{3}, c_{4}\right\}, F\left(e_{4}\right)=\left\{c_{1}, c_{7}\right\}$,
$F\left(e_{5}\right)=\left\{c_{5}, c_{6}, c_{7}\right\}, F\left(e_{7}\right)=\left\{c_{1}, c_{2}, c_{7}\right\}$,
$F\left(e_{8}\right)=\left\{c_{3}, c_{4}, c_{5}, c_{6}\right\}$.
Let Mr. X has priority ranking in getting manual, diesel, new, second hand and light color cars. According to this priority, we can define a soft set relation on ( $F, E$ ) as follows:
$R=\left\{F\left(e_{1}\right) \times F\left(e_{5}\right), F\left(e_{7}\right) \times F\left(e_{5}\right), F\left(e_{8}\right) \times F\left(e_{5}\right)\right.$,
$F\left(e_{3}\right) \times F\left(e_{5}\right), F\left(e_{1}\right) \times F\left(e_{1}\right), F\left(e_{8}\right) \times F\left(e_{1}\right), F\left(e_{3}\right)$
$\left.\times F\left(e_{1}\right), F\left(e_{8}\right) \times F\left(e_{7}\right), F\left(e_{3}\right) \times F\left(e_{7}\right), F\left(e_{3}\right) \times F\left(e_{8}\right)\right\}$.
Graph of this relation $R$ is given in Graph 3.


Graph 3. Graph representation of relation $R$.
Let's apply the above method to this problem. Step 1: Counting paths for each node:

$$
\begin{array}{ll}
F\left(e_{1}\right) & (3,1) \\
F\left(e_{3}\right) & (0,4) \\
F\left(e_{5}\right) & (4,0) \\
F\left(e_{7}\right) & (2,2) \\
F\left(e_{8}\right) & (1,3)
\end{array}
$$

Step 2: Images of $F\left(e_{i}\right) \mathrm{s}$ and the existence vectors:

$$
\begin{array}{lr}
F\left(e_{1}\right)=\left\{c_{1}, c_{3}, c_{5}\right\} & (1,0,1,0,1,0,0) \\
F\left(e_{3}\right)=\left\{c_{2}, c_{3}, c_{4}\right\} & (0,1,1,1,0,0,0) \\
F\left(e_{5}\right)=\left\{c_{5}, c_{6}, c_{7}\right\} & (0,0,0,0,1,1,1) \\
F\left(e_{7}\right)=\left\{c_{1}, c_{2}, c_{7}\right\} & (1,1,0,0,0,0,1) \\
F\left(e_{8}\right)=\left\{c_{3}, c_{4}, c_{5}, c_{6}\right\} & (0,0,1,1,1,1,0)
\end{array}
$$

Step 3: Change of images:
$F\left(e_{1}\right) \quad(4,0,4,0,4,0,0) \quad(2,0,2,0,2,0,0)$
$F\left(e_{3}\right) \quad(0,1,1,1,0,0,0) \quad(0,5,5,5,0,0,0)$
$F\left(e_{5}\right) \quad(0,0,0,0,5,5,5) \quad(0,0,0,0,5,5,5)$
$F\left(e_{7}\right) \quad(3,3,0,0,0,0,3) \quad(3,3,0,0,0,0,3)$
$F\left(e_{8}\right) \quad(0,0,2,2,2,2,0) \quad(0,0,4,4,4,4,0)$
Step 4: Total incoming and outgoing values $c_{1}, c_{2}, c_{3}, c_{4}, c_{5}, c_{6}$ and $c_{7}$ are (7,5), (4,8), (7,11), $(3,9),(11,7),(7,5)$ and $(8,4)$, respectively. Hence the car will be $c_{5}$ according to our algorithm.
Same results obtained in the above steps are summarized in Table 2.

Table 2. Summarized Algorithm for Example 3.7.

|  | $c_{1}$ | $c_{2}$ | $C_{3}$ | $C_{4}$ | $c_{5}$ | $c_{6}$ | $C_{7}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{3} F\left(e_{1}\right)_{1}$ | ${ }_{4} 1_{2}$ | ${ }_{0} 0_{0}$ | ${ }_{4} 1_{2}$ | ${ }_{0} 0_{0}$ | ${ }_{4} 1_{2}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ |
| ${ }_{0} F\left(e_{2}\right)_{4}$ | ${ }_{0} 0_{0}$ | ${ }_{1} 1_{5}$ | ${ }_{1} 1_{5}$ | ${ }_{1} 1_{5}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ |
| ${ }_{4} F\left(e_{5}\right)_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{5} 1_{1}$ | ${ }_{5} 1_{1}$ | ${ }_{5} 1_{1}$ |
| ${ }_{2} F\left(e_{7}\right)_{2}$ | ${ }_{3} 1_{3}$ | ${ }_{3} 1_{3}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{3} 1_{3}$ |
| ${ }_{1} F\left(e_{8}\right)_{3}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0$ | ${ }_{1} 1_{4}$ | ${ }_{1} 1_{4}$ | ${ }_{1} 1_{4}$ | ${ }_{1} 1_{4}$ | ${ }_{0} 0_{0}$ |
|  | $(7,5)$ | $(4,8)$ | $(7,11)$ | $(3,9)$ | $(11,7)$ | $(7,5)$ | $(8,4)$ |

The following example is about Information Systems (IS) that was given by Jiang, et. al. (2011). We have successfully applied our designed Decision Making method to this example.
Example 3.8: Suppose that there are six papers $p_{1}, p_{2}, p_{3}, p_{4}, p_{5}, p_{6}$ in IS (Table 3) and four keywords $a_{1}, a_{2}, a_{3}, a_{4}$ stand for "keyword 1= Semantic web", "keyword 2"=Description logics", "keyword 3=Web Ontology Language", and "keyword 4=Reasoning Rule", respectively.

Table 3. Information system (IS).

| $\mathbf{U}$ | $a_{1}$ | $a_{2}$ | $a_{3}$ | $a_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $p_{1}$ | DLs | Soft Sets | Ontology | Decision <br> Making |
| $p_{2}$ | Data <br> mining | Data <br> bases | Machine <br> learning | Rule |
| $p_{3}$ | Ontology | Rule | DLs | OWL |
| $p_{4}$ | OWL | Ontology | KR | DLs |
| $p_{5}$ | LP | ASP | First Order <br> Logic | Prolog |
| $p_{6}$ | ASP | Prolog | LP | KR |

Assume that Mr. X wants to obtain a closely related research paper in the area of "Semantic web" using the following query:
"Keyword 1"=Semantic web, "keyword 2"=Description logics, "keyword 3"=Web Ontology Language, and "keyword 4" =Reasoning Rule.

It is reported Jiang, et. al. (2011) that the approximately resultant soft set, where $\left\{a_{1}, a_{2}, a_{3}, a_{4}\right\}$ is the parameter set and $\left\{p_{1}, p_{2}, p_{3}, p_{4}, p_{5}, p_{6}\right\}$ is the universal set, is obtained from certain semantic relations between the information system and the query as follows:

$$
\begin{aligned}
& F\left(a_{1}\right)=\left\{p_{1}, p_{3}, p_{4}, p_{5}, p_{6}\right\}, F\left(a_{2}\right)=\left\{p_{3}, p_{4}, p_{5}\right\}, \\
& F\left(a_{3}\right)=\left\{p_{1}, p_{3}, p_{4}, p_{5}, p_{6}\right\}, F\left(a_{4}\right)=\left\{p_{2}, p_{3}, p_{4}, p_{6}\right\}
\end{aligned}
$$

Now, let's find the choice object for Mr. X using our Decision Making method.

A soft set relation $R$ can be obtained by given query as:

$$
\begin{aligned}
& <=R=\left\{F\left(a_{1}\right) \times F\left(a_{2}\right), F\left(a_{1}\right) \times F\left(a_{3}\right), F\left(a_{1}\right) \times F\left(a_{4}\right),\right. \\
& \left.F\left(a_{2}\right) \times F\left(a_{3}\right), F\left(a_{2}\right) \times F\left(a_{4}\right), F\left(a_{3}\right) \times F\left(a_{4}\right)\right\} .
\end{aligned}
$$

Hence, the graph representation of $R$ is:


Graph 4. Graph representation of relation $R$.
After applying our Decision Making method to this example, we obtain the choice objects as $p_{3}$ and $p_{4}$ (Table 4). These choice objects are the same as stated by Jiang, et. al. (2011).

Table 4. Summarized Algorithm for Example 3.8.

|  | $p_{1}$ | $p_{2}$ | $p_{3}$ | $p_{4}$ | $p_{5}$ | $p_{6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{0} F\left(e_{1}\right)_{3}$ | ${ }_{1} 1_{4}$ | ${ }_{0} 0_{0}$ | ${ }_{1} 1_{4}$ | ${ }_{1} 1_{4}$ | ${ }_{1} 1_{4}$ | ${ }_{1} 1_{4}$ |
| ${ }_{1} F\left(e_{2}\right)_{2}$ | ${ }_{0} 0_{0}$ | ${ }_{0} 0_{0}$ | ${ }_{2} 1_{3}$ | ${ }_{2} 1_{3}$ | ${ }_{2} 1_{3}$ | ${ }_{0} 0_{0}$ |
| ${ }_{2} F\left(e_{3}\right)_{1}$ | ${ }_{3} 1_{2}$ | ${ }_{0} 0_{0}$ | ${ }_{3} 1_{2}$ | ${ }_{3} 1_{2}$ | ${ }_{3} 1_{2}$ | ${ }_{3} 1_{2}$ |
| ${ }_{3} F\left(e_{4}\right)_{3}$ | ${ }_{0} 0_{0}$ | ${ }_{4} 1_{4}$ | ${ }_{4} 1_{4}$ | ${ }_{4} 1_{4}$ | ${ }_{0} 0_{0}$ | ${ }_{4} 1_{4}$ |
|  | $(4,6)$ | $(4,4)$ | $(\mathbf{1 0 , 1 3})$ | $(\mathbf{1 0 , 1 3})$ | $(6,9)$ | $(8,10)$ |

## 4 CONCLUSIONS

IN this study, we have designed a Decision Making Method by using graph representation of soft set relations that is a new perspective for the soft set relations. By utilizing this method, complex social life
problems in different areas can be solved effectively. This method can be also used in various decision making methods including artificial intelligence.

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