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A numerical approach to solve the model for HIV infection of CD4$^+$T cells

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ABSTRACT

In this study, we will obtain the approximate solutions of the HIV infection model of CD4$^+$*T* by developing the Bessel collocation method. This model corresponds to a class of nonlinear ordinary differential equation systems. Proposed scheme consists of reducing the problem to a nonlinear algebraic equation system by expanding the approximate solutions by means of the Bessel polynomials with unknown coefficients. The unknown coefficients of the Bessel polynomials are computed using the matrix operations of derivatives together with the collocation method. The reliability and efficiency of the proposed approach are demonstrated in the different time intervals by a numerical example. All computations have been made with the aid of a computer code written in Maple 9.

1. Introduction

In this study, we consider the HIV infection model of CD4$^+$*T* cells is examined [1]. This model is given by the components of the basic three-component model are the concentration of susceptible CD4$^+$*T* cells, CD4$^+$*T* cells infected by the HIV viruses and free HIV virus particles in the blood. CD4$^+$*T* cells are also called as leukocytes or *T* helper cells. These with order cells in human immunity systems fight against diseases. HIV use cells in order to propagate. In a healthy person, the number of CD4$^+$*T* cells is $\frac{800}{1200}$ mm^3. This model is characterized by a system of the nonlinear differential equations

$$\begin{cases} \frac{dT}{dt} = q - \alpha T + rT\left(1 - \frac{T+I}{T_{\max}}\right) - kVT \\ \frac{dI}{dt} = kVT - \beta I \\ \frac{dV}{dt} = \mu\beta I - \gamma V \end{cases}, \quad T(0) = r_1, \quad I(0) = r_2, \quad V(0) = r_3, \quad 0 \leqslant t \leqslant R < \infty. \tag{1}$$

Here, R is any positive constant, $T(t)$, $I(t)$ and $V(t)$ show the concentration of susceptible CD4$^+$*T* cells, CD4$^+$*T* cells infected by the HIV viruses and free HIV virus particles in the blood, respectively, α, β and γ denote natural turnover rates of uninfected *T* cells, infected *T* cells and virus particles, respectively, $\left(1 - \frac{T+I}{T_{\max}}\right)$ describes the logistic growth of the healthy CD4$^+$*T* cells, and proliferation of infected CD4$^+$*T* cells is neglected. For $k > 0$ is the infection rate, the term KVT describes the incidence of HIV infection of healthy CD4$^+$*T* cells. Each infected CD4$^+$*T* cell is assumed to produce μ virus particles during its lifetime, including any of its daughter cells. The body is believed to produce CD4$^+$*T* cells from precursors in the bone marrow and thymus at a constant rate q. *T* cells multiply through mitosis with a rate r when *T* cells are stimulated by antigen or mitogen. $T_{\max}$ denotes the maximum CD4$^+$*T* cell concentration in the body [2–5]. In this article, we set $q = 0.1$, $\alpha = 0.02$, $\beta = 0.3$, $r = 3$, $\gamma = 2.4$, $k = 0.0027$, $T_{\max} = 1500$, $\mu = 10$.

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For solving numerically a model for HIV infection of CD4^+T cells, Ongun [6] have applied the Laplace adomian decomposition method, Merdan have used the homotopy perturbation method [7] and Merdan et al. have applied the Padé approximate and the modified variational iteration method [8].

Recently, Yüzbaşı et al. [9–15] have studied the Bessel collocation method for the approximate solutions of the Lane–Emden differential, neutral delay differential, pantograph, Volterra integro-differential and Fredholm integro-differential-difference equations, Fredholm integro-differential equation systems and the pollution model of a system of lakes, and also Yüzbaşı [16–18] have developed the Bessel collocation method for solving numerically the singular differential-difference equations, a class of the nonlinear Lane–Emden type equations arising in astrophysics and the continuous population models for single and interacting species.

In this paper, we will obtain the approximate solutions of model (1) by developing the Bessel collocation method studied in [1–10]. Our purpose is to find approximate solutions of model (1) in the truncated Bessel series forms

$$T(t)=\sum_{n=0}^{N}a_{1,n}J_n(t),\quad I(t)=\sum_{n=0}^{N}a_{2,n}J_n(t)\quad\text{and}\quad V(t)=\sum_{n=0}^{N}a_{3,n}J_n(t) \tag{2}$$

so that $a_{1,n}$, $a_{2,n}$ and $a_{3,n}$ ($n = 0,1,2,\ldots,N$) are the unknown Bessel coefficients and $J_n(t)$, $n = 0,1,2,\ldots,N$ are the Bessel polynomials of first kind defined by

$$J_n(t)=\sum_{k=0}^{[\![\frac{N-n}{2}]\!]}\frac{(-1)^k}{k!(k+n)!}\left(\frac{t}{2}\right)^{2k+n},\quad n\in\mathbb{N},\quad 0\leqslant t<\infty. \tag{3}$$

2. Method for solution

Firstly, let us show model (1) in the form

$$\begin{cases}\frac{dT}{dt}=q+(r-\alpha)T-\frac{r}{T_{\max}}T^2-\frac{r}{T_{\max}}TI-kVT\\ \frac{dI}{dt}=kVT-\beta I\\ \frac{dV}{dt}=\mu\beta I-\gamma V\end{cases}. \tag{4}$$

We consider the approximate solutions $T(t)$, $I(t)$ and $V(t)$ given by (2) of the system (4).

Now, let us write the matrix forms of the solution functions defined in relation (2) as

$$T(t)=\mathbf{J}(t)\mathbf{A}_1,\quad I(t)=\mathbf{J}(t)\mathbf{A}_2\quad\text{and}\quad V(t)=\mathbf{J}(t)\mathbf{A}_3 \tag{5}$$

where

$$\mathbf{J}(t)=[J_0(t)\quad J_1(t)\quad\cdots\quad J_N(t)],\quad \mathbf{A}_1=[a_{1,0}\quad a_{1,1}\quad\cdots\quad a_{1,N}]^T,\quad \mathbf{A}_2=[a_{2,0}\quad a_{2,1}\quad\cdots\quad a_{2,N}]^T$$

and $\mathbf{A}_3=[a_{3,0}\quad a_{3,1}\quad\cdots\quad a_{3,N}]^T$.

Also, the relations given by (5) can be written in matrix forms

$$T(t)=\mathbf{T}(t)\mathbf{D}^{\mathbf{T}}\mathbf{A}_1,\quad I(t)=\mathbf{T}(t)\mathbf{D}^{\mathbf{T}}\mathbf{A}_2\quad\text{and}\quad V(t)=\mathbf{T}(t)\mathbf{D}^{\mathbf{T}}\mathbf{A}_3 \tag{6}$$

so that $\mathbf{T}(t)=[1\quad t\quad t^2\quad\cdots\quad t^N]$ and if N is odd,

$$\mathbf{D}=\begin{bmatrix}\frac{1}{0!0!2^0} & 0 & \frac{-1}{1!1!2^2} & \cdots & \frac{(-1)^{\frac{N-1}{2}}}{\left(\frac{N-1}{2}\right)!\left(\frac{N-1}{2}\right)!2^{N-1}} & 0\\ 0 & \frac{1}{0!1!2^1} & 0 & \cdots & 0 & \frac{(-1)^{\frac{N-1}{2}}}{\left(\frac{N-1}{2}\right)!\left(\frac{N+1}{2}\right)!2^{N}}\\ 0 & 0 & \frac{1}{0!2!2^2} & \cdots & \frac{(-1)^{\frac{N-3}{2}}}{\left(\frac{N-3}{2}\right)!\left(\frac{N+1}{2}\right)!2^{N-1}} & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & \frac{1}{0!(N-1)!2^{N-1}} & 0\\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{0!N!2^N}\end{bmatrix}_{(N+1)\times(N+1)},$$

if N is even,

$$\mathbf{D}=\begin{bmatrix} \frac{1}{0!0!2^0} & 0 & \frac{-1}{1!1!2^2} & \cdots & 0 & \frac{(-1)^{\frac{N}{2}}}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!2^N} \\ 0 & \frac{1}{0!1!2^1} & 0 & \cdots & \frac{(-1)^{\frac{N-2}{2}}}{\left(\frac{N-2}{2}\right)!\left(\frac{N}{2}\right)!2^{N-1}} & 0 \\ 0 & 0 & \frac{1}{0!2!2^2} & \cdots & 0 & \frac{(-1)^{\frac{N-2}{2}}}{\left(\frac{N-2}{2}\right)!\left(\frac{N+2}{2}\right)!2^N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{0!(N-1)!2^{N-1}} & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{0!N!2^N} \end{bmatrix}_{(N+1)\times(N+1)}.$$

On the other hand, the relation between the matrix $\mathbf{T}(t)$ and its derivative $\mathbf{T}^{(1)}(t)$ is

$$\mathbf{T}^{(1)}(t)=\mathbf{T}(t)\mathbf{B}^T \tag{7}$$

where

$$\mathbf{B}^T=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & N \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

By aid of the relations (6) and (7), we have recurrence relations

$$T^{(1)}(t)=\mathbf{T}(t)\mathbf{B}^T\mathbf{D}^{\mathbf{T}}\mathbf{A}_1,\quad I^{(1)}(t)=\mathbf{T}(t)\mathbf{B}^T\mathbf{D}^{\mathbf{T}}\mathbf{A}_2\quad \text{and}\quad V^{(1)}(t)=\mathbf{T}(t)\mathbf{B}^T\mathbf{D}^{\mathbf{T}}\mathbf{A}_3. \tag{8}$$

Thus, we can express the matrices $\mathbf{y}(t)$ and $\mathbf{y}^{(1)}(t)$ as follows:

$$\mathbf{y}(t)=\overline{\mathbf{T}}(t)\overline{\mathbf{D}}\mathbf{A}\quad \text{and}\quad \mathbf{y}^{(1)}(t)=\overline{\mathbf{T}}(t)\overline{\mathbf{B}}\overline{\mathbf{D}}\mathbf{A} \tag{9}$$

so that $\mathbf{y}(t)=[\,T(t)\quad I(t)\quad V(t)\,]^T$, $\quad\mathbf{y}^{(1)}(t)=[\,T^{(1)}(t)\quad I^{(1)}(t)\quad V^{(1)}(t)\,]^T$,

$$\overline{\mathbf{T}}(t)=\begin{bmatrix} \mathbf{T}(t) & 0 & 0 \\ 0 & \mathbf{T}(t) & 0 \\ 0 & 0 & \mathbf{T}(t) \end{bmatrix}, \overline{\mathbf{D}}=\begin{bmatrix} \mathbf{D}^T & 0 & 0 \\ 0 & \mathbf{D}^T & 0 \\ 0 & 0 & \mathbf{D}^T \end{bmatrix}, \overline{\mathbf{B}}=\begin{bmatrix} \mathbf{B}^T & 0 & 0 \\ 0 & \mathbf{B}^T & 0 \\ 0 & 0 & \mathbf{B}^T \end{bmatrix}\quad \text{and}\quad \mathbf{A}=\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \end{bmatrix}.$$

Now, we can show the system (4) with the matrix form

$$\mathbf{y}^{(1)}(t)-\mathbf{P}\mathbf{y}(t)-\mathbf{M}\bar{\mathbf{y}}(t)\mathbf{y}(t)-\mathbf{L}\mathbf{y}_{1,2}(t)-\mathbf{K}\mathbf{y}_{1,3}(t)=\mathbf{q} \tag{10}$$

where

$$\mathbf{q}=\begin{bmatrix} q \\ 0 \\ 0 \end{bmatrix},\quad \mathbf{P}=\begin{bmatrix} r-\alpha & 0 & 0 \\ 0 & -\beta & 0 \\ 0 & \mu\beta & -\gamma \end{bmatrix},\quad \mathbf{y}(\mathbf{t})=\begin{bmatrix} T(t) \\ I(t) \\ V(t) \end{bmatrix},\quad \mathbf{M}=\begin{bmatrix} -r/T_{\max} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad \bar{\mathbf{y}}(t)=\begin{bmatrix} T(t) & 0 & 0 \\ 0 & I(t) & 0 \\ 0 & 0 & V(t) \end{bmatrix},$$

$$\mathbf{L}=\begin{bmatrix} -r/T_{\max} \\ 0 \\ 0 \end{bmatrix},\quad \mathbf{y}_{1,2}(t)=[T(t)I(t)],\quad \mathbf{K}=\begin{bmatrix} -k \\ k \\ 0 \end{bmatrix}\quad \text{and}\quad \mathbf{y}_{1,3}(t)=[V(t)T(t)].$$

In Eq. (10), we use the collocation points

$$t_i=\frac{R}{N}i,\quad i=0,1,\ldots,N,\quad (0\leqslant t\leqslant R), \tag{11}$$

and thus we obtain a system of the matrix equations

$$\mathbf{y}^{(1)}(t_s)-\mathbf{P}\mathbf{y}(t_s)-\mathbf{M}\bar{\mathbf{y}}(t_s)\mathbf{y}(t_s)-\mathbf{L}\mathbf{y}_{1,2}(t_s)-\mathbf{K}\mathbf{y}_{1,3}(t_s)=\mathbf{q}$$

or briefly the fundamental matrix equation

$$\mathbf{Y}^{(1)}-\overline{\mathbf{P}}\mathbf{Y}-\overline{\mathbf{M}}\overline{\mathbf{Y}}\mathbf{Y}-\overline{\mathbf{L}}\widetilde{\overline{\mathbf{Y}}}-\overline{\mathbf{K}}\overline{\overline{\mathbf{Y}}}=\mathbf{Q} \tag{12}$$

where

$$\mathbf{Y}^{(1)}=\begin{bmatrix}\mathbf{y}^{(1)}(t_0)\\ \mathbf{y}^{(1)}(t_1)\\ \vdots\\ \mathbf{y}^{(1)}(t_N)\end{bmatrix},\quad \overline{\mathbf{P}}=\begin{bmatrix}\mathbf{P}&0&\cdots&0\\0&\mathbf{P}&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{P}\end{bmatrix}_{(N+1)\times(N+1)},\quad \mathbf{Y}=\begin{bmatrix}\mathbf{y}(t_0)\\ \mathbf{y}(t_1)\\ \vdots\\ \mathbf{y}(t_N)\end{bmatrix},\quad \mathbf{Q}=\begin{bmatrix}\mathbf{q}\\ \mathbf{q}\\ \vdots\\ \mathbf{q}\end{bmatrix}_{(N+1)\times 1},$$

$$\overline{\mathbf{M}}=\begin{bmatrix}\mathbf{M}&0&\cdots&0\\0&\mathbf{M}&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{M}\end{bmatrix}_{(N+1)\times(N+1)},\quad \overline{\mathbf{Y}}=\begin{bmatrix}\bar{\mathbf{y}}(t_0)&0&\cdots&0\\0&\bar{\mathbf{y}}(t_1)&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\bar{\mathbf{y}}(t_N)\end{bmatrix},\quad \widetilde{\widetilde{\mathbf{Y}}}=\begin{bmatrix}\mathbf{y}_{1,2}(t_0)\\ \mathbf{y}_{1,2}(t_1)\\ \vdots\\ \mathbf{y}_{1,2}(t_N)\end{bmatrix},$$

$$\overline{\mathbf{L}}=\begin{bmatrix}\mathbf{L}&0&\cdots&0\\0&\mathbf{L}&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{L}\end{bmatrix}_{(N+1)\times(N+1)},\quad \overline{\mathbf{K}}=\begin{bmatrix}\mathbf{K}&0&\cdots&0\\0&\mathbf{K}&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{K}\end{bmatrix}_{(N+1)\times(N+1)}\quad\text{and}\quad \overline{\overline{\mathbf{Y}}}=\begin{bmatrix}\mathbf{y}_{1,3}(t_0)\\ \mathbf{y}_{1,3}(t_1)\\ \vdots\\ \mathbf{y}_{1,3}(t_N)\end{bmatrix}.$$

By using relations (9) and the collocation points (11), we have

$$\mathbf{y}(t_s)=\overline{\mathbf{T}}(t_s)\overline{\mathbf{D}}\mathbf{A}\quad\text{and}\quad \mathbf{y}^{(1)}(t_s)=\overline{\mathbf{T}}(t_s)\overline{\mathbf{B}}\overline{\mathbf{D}}\mathbf{A}$$

which can be written as

$$\mathbf{Y}=\mathbf{T}\overline{\mathbf{D}}\mathbf{A}\quad\text{and}\quad \mathbf{Y}^{(1)}=\mathbf{T}\overline{\mathbf{B}}\overline{\mathbf{D}}\mathbf{A} \tag{13}$$

where

$$\mathbf{T}=\begin{bmatrix}\overline{\mathbf{T}}(t_0)&\overline{\mathbf{T}}(t_1)&\ldots&\overline{\mathbf{T}}(t_N)\end{bmatrix}^T,\quad \overline{\mathbf{T}}(t_s)=\begin{bmatrix}\mathbf{T}(t_s)&0&0\\0&\mathbf{T}(t_s)&0\\0&0&\mathbf{T}(t_s)\end{bmatrix}\quad\text{and}\quad s=0,1,\ldots,N.$$

By aid of the collocation points (11) and the matrix $\bar{\mathbf{y}}(t)$ given in Eq. (10), we have

$$\overline{\mathbf{Y}}=\begin{bmatrix}\bar{\mathbf{y}}(t_0)&0&\cdots&0\\0&\bar{\mathbf{y}}(t_1)&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\bar{\mathbf{y}}(t_N)\end{bmatrix}=\begin{bmatrix}\overline{\mathbf{T}}(t_0)\overline{\mathbf{D}\mathbf{A}}&0&\cdots&0\\0&\overline{\mathbf{T}}(t_1)\overline{\mathbf{D}\mathbf{A}}&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\overline{\mathbf{T}}(t_N)\overline{\mathbf{D}\mathbf{A}}\end{bmatrix}=\overline{\mathbf{T}}\overline{\overline{\mathbf{D}\mathbf{A}}} \tag{14}$$

so that

$$\overline{\mathbf{T}}=\begin{bmatrix}\overline{\mathbf{T}}(t_0)&0&\cdots&0\\0&\overline{\mathbf{T}}(t_1)&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\overline{\mathbf{T}}(t_N)\end{bmatrix},\quad \overline{\mathbf{T}}(t)=\begin{bmatrix}\mathbf{T}(t)&0&0\\0&\mathbf{T}(t)&0\\0&0&\mathbf{T}(t)\end{bmatrix},\quad \overline{\overline{\mathbf{D}}}=\begin{bmatrix}\overline{\mathbf{D}}&0&\cdots&0\\0&\overline{\mathbf{D}}&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\overline{\mathbf{D}}\end{bmatrix}_{(N+1)\times(N+1)},$$

$$\overline{\mathbf{D}}=\begin{bmatrix}\mathbf{D}^T&0&0\\0&\mathbf{D}^T&0\\0&0&\mathbf{D}^T\end{bmatrix},\quad \overline{\overline{\mathbf{A}}}=\begin{bmatrix}\widetilde{\mathbf{A}}&0&\cdots&0\\0&\widetilde{\mathbf{A}}&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\widetilde{\mathbf{A}}\end{bmatrix}_{(N+1)\times(N+1)}\quad\text{and}\quad \widetilde{\mathbf{A}}=\begin{bmatrix}\mathbf{A}_1&0&0\\0&\mathbf{A}_2&0\\0&0&\mathbf{A}_3\end{bmatrix}.$$

Similarly, substituting the collocation points (11) into the $y_{1,2}(t)$ and $y_{1,3}(t)$ given Eq. (10), we obtain the matrix representation

$$\widetilde{\widetilde{\mathbf{Y}}}=\begin{bmatrix}\mathbf{y}_{1,2}(t_0)\\ \mathbf{y}_{1,2}(t_1)\\ \vdots\\ \mathbf{y}_{1,2}(t_N)\end{bmatrix}=\begin{bmatrix}I(t_0)T(t_0)\\ I(t_1)T(t_1)\\ \vdots\\ I(t_N)T(t_N)\end{bmatrix}=\overline{\mathbf{I}}\overline{\mathbf{T}}\quad\text{and}\quad \overline{\overline{\mathbf{Y}}}=\begin{bmatrix}\mathbf{y}_{1,3}(t_0)\\ \mathbf{y}_{1,3}(t_1)\\ \vdots\\ \mathbf{y}_{1,3}(t_N)\end{bmatrix}=\begin{bmatrix}V(t_0)T(t_0)\\ V(t_1)T(t_1)\\ \vdots\\ V(t_N)T(t_N)\end{bmatrix}=\overline{\mathbf{V}}\overline{\mathbf{T}} \tag{15}$$

where

$$\overline{\mathbf{I}}=\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}\overline{\mathbf{A}}_2,\quad \overline{\mathbf{V}}=\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}\overline{\mathbf{A}}_3\quad\text{and}\quad \overline{\mathbf{T}}=\widetilde{\widetilde{\mathbf{T}}}\,\widetilde{\widetilde{\mathbf{D}}}\mathbf{A} \tag{16}$$

so that

$$\widetilde{\mathbf{T}}=\begin{bmatrix}\mathbf{T}(t_0)&0&\cdots&0\\0&\mathbf{T}(t_1)&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{T}(t_N)\end{bmatrix},\quad \widetilde{\mathbf{D}}=\begin{bmatrix}\mathbf{D}^T&0&\cdots&0\\0&\mathbf{D}^T&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{D}^T\end{bmatrix}_{(N+1)\times(N+1)},$$

$$\overline{\mathbf{A}}_2=\begin{bmatrix}\mathbf{A}_2&0&\cdots&0\\0&\mathbf{A}_2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{A}_2\end{bmatrix}_{(N+1)\times(N+1)},\quad \overline{\mathbf{A}}_3=\begin{bmatrix}\mathbf{A}_3&0&\cdots&0\\0&\mathbf{A}_3&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{A}_3\end{bmatrix}_{(N+1)\times(N+1)},\quad \tilde{\tilde{\mathbf{T}}}=\begin{bmatrix}\mathbf{T}(t_0)\\\mathbf{T}(t_1)\\\vdots\\\mathbf{T}(t_N)\end{bmatrix},$$

$$\tilde{\tilde{\mathbf{D}}}=[\mathbf{D}^T\quad\mathbf{0}\quad\mathbf{0}],\quad \mathbf{0}=\begin{bmatrix}0&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0\end{bmatrix}_{(N+1)\times(N+1)}\quad\text{and}\quad \mathbf{A}=\begin{bmatrix}\mathbf{A}_1\\\mathbf{A}_2\\\mathbf{A}_3\end{bmatrix}.$$

Substituting relations (13)–(16) into Eq. (12), we have the fundamental matrix equation

$$\left\{\mathbf{T}\overline{\mathbf{B}}\overline{\mathbf{D}}-\overline{\mathbf{P}}\mathbf{T}\overline{\mathbf{D}}-\overline{\mathbf{M}}\widetilde{\mathbf{T}}\overline{\overline{\mathbf{D}}}\overline{\mathbf{A}}\mathbf{T}\overline{\mathbf{D}}-\overline{\mathbf{L}}\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}\overline{\mathbf{A}}_2\tilde{\tilde{\mathbf{T}}}\tilde{\tilde{\mathbf{D}}}-\overline{\mathbf{K}}\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}\overline{\mathbf{A}}_3\tilde{\tilde{\mathbf{T}}}\tilde{\tilde{\mathbf{D}}}\right\}\mathbf{A}=\mathbf{Q}. \tag{17}$$

We can write briefly Eq. (17) in the form

$$\mathbf{WA}=\mathbf{Q}\ \text{or}\ [\mathbf{W};\mathbf{Q}];\mathbf{W}=\mathbf{T}\overline{\mathbf{B}}\overline{\mathbf{D}}-\overline{\mathbf{P}}\mathbf{T}\overline{\mathbf{D}}-\overline{\mathbf{M}}\widetilde{\mathbf{T}}\overline{\overline{\mathbf{D}}}\overline{\mathbf{A}}\mathbf{T}\overline{\mathbf{D}}-\overline{\mathbf{L}}\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}\overline{\mathbf{A}}_2\tilde{\tilde{\mathbf{T}}}\tilde{\tilde{\mathbf{D}}}-\overline{\mathbf{K}}\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}\overline{\mathbf{A}}_3\tilde{\tilde{\mathbf{T}}}\tilde{\tilde{\mathbf{D}}} \tag{18}$$

which corresponds to a system of the $3(N+1)$ nonlinear algebraic equations with the unknown Bessel coefficients $a_{1,n}$, $a_{2,n}$ and $a_{3,n}$, $(n=0,1,2,\ldots,N)$.

By aid of the relation (9), the matrix form for conditions given in model (1) can be written as

$$\mathbf{UA}=[\overline{\mathbf{R}}]\ \text{or}\ [\mathbf{U};\overline{\mathbf{R}}] \tag{19}$$

so that $\mathbf{U}=\overline{\mathbf{T}}(0)\overline{\mathbf{D}}\mathbf{A}$ and $\overline{\mathbf{R}}=\begin{bmatrix}r_1\\r_2\\r_3\end{bmatrix}$. Consequently, by replacing the rows of the matrix $[\mathbf{U};\overline{\mathbf{R}}]$ by three rows of the augmented matrix $[\mathbf{W};\mathbf{Q}]$, we have the new augmented matrix

$$[\widetilde{\mathbf{W}};\widetilde{\mathbf{Q}}]\ \text{or}\ \widetilde{\mathbf{W}}\mathbf{A}=\widetilde{\mathbf{Q}} \tag{20}$$

which is a nonlinear algebraic system. The unknown the Bessel coefficients can be computed by solving this system. The unknown Bessel coefficients $a_{i,0},a_{i,1},\ldots,a_{i,N}$, $(i=1,2,3)$ is substituted in Eq. (2). Thus we obtain the Bessel polynomial solutions

$$T_N(t)=\sum_{n=0}^{N}a_{1,n}J_n(t),\quad I_N(t)=\sum_{n=0}^{N}a_{2,n}J_n(t)\quad\text{and}\quad V_N(t)=\sum_{n=0}^{N}a_{3,n}J_n(t).$$

We can easily check the accuracy of this solutions as follows:

Since the truncated Bessel series (2) are approximate solutions of the system (1), when the function $T_N(t)$, $I_N(t)$, $V_N(t)$ and theirs derivatives are substituted in system (1), the resulting equation must be satisfied approximately; that is, for $t=t_q\in[0,R]$ $q=0,1,2,\ldots$

$$\begin{cases}E_{1,N}(t_q)=\left|T_N^{(1)}(t_q)-q+\alpha T_N(t_q)-rT_N(t_q)\left(1-\frac{T_N(t_q)+I_N(t_q)}{T_{\max}}\right)+kV_N(t_q)T_N(t_q)\right|\cong 0,\\ E_{2,N}(t_q)=\left|I_N^{(1)}(t_q)-kV_N(t_q)T_N(t_q)+\beta I_N(t_q)\right|\cong 0,\\ E_{3,N}(t_q)=\left|V_N^{(1)}(t_q)-\mu\beta I_N(t_q)+\gamma V_N(t_q)\right|\cong 0\end{cases} \tag{21}$$

and $E_{i,N}(t_q)\leqslant 10^{-k_q}$, $i=1,2,3$ (k_q positive integer).

If $\max 10^{-k_q}=10^{-k}$ (k positive integer) is prescribed, then the truncation limit N is increased until the difference $E_{i,N}(t_q)$, $(i=1,2,3)$ at each of the points becomes smaller than the prescribed 10^{-k}, see [9–22].

3. Numerical applications

In this section, we have applied the method presented for model (1) with the initial conditions $T(0)=0.1$, $I(0)=0$, $V(0)=0.1$ in the intervals $0\leqslant t\leqslant 1$, $0\leqslant t\leqslant 2$, $0\leqslant t\leqslant 5$, $0\leqslant t\leqslant 10$, $0\leqslant t\leqslant 15$ and $0\leqslant t\leqslant 20$. We get the approximate solutions by applying the present method for $N=3$, 8 in the above intervals. By using the present method for $N=3$, 8 in interval $0\leqslant t\leqslant 1$, we obtain the approximate solutions, respectively,

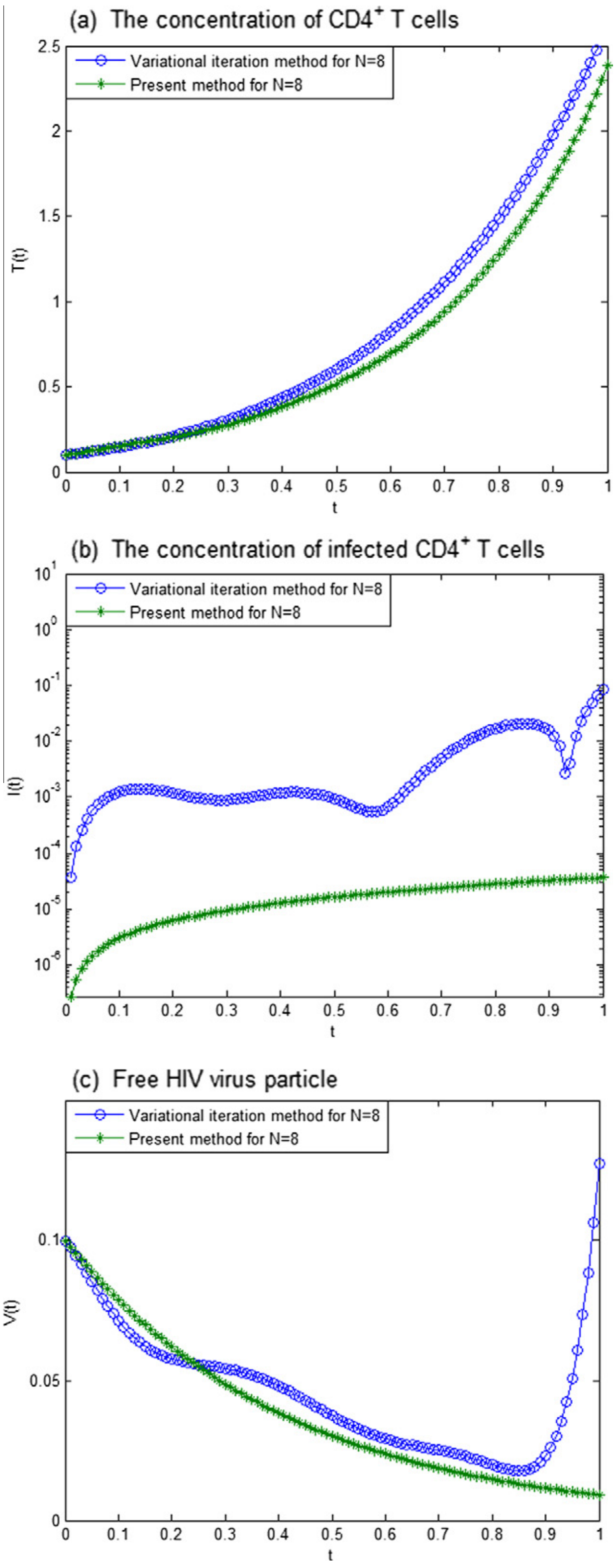

Fig. 1. For $N = 8$ in the interval $0 \leqslant t \leqslant 1$, (**a**) comparison of the approximate solutions $T_N(t)$, (**b**) comparison of the approximate solutions $I_N(t)$ and (**c**) comparison of the approximate solutions $V_N(t)$.

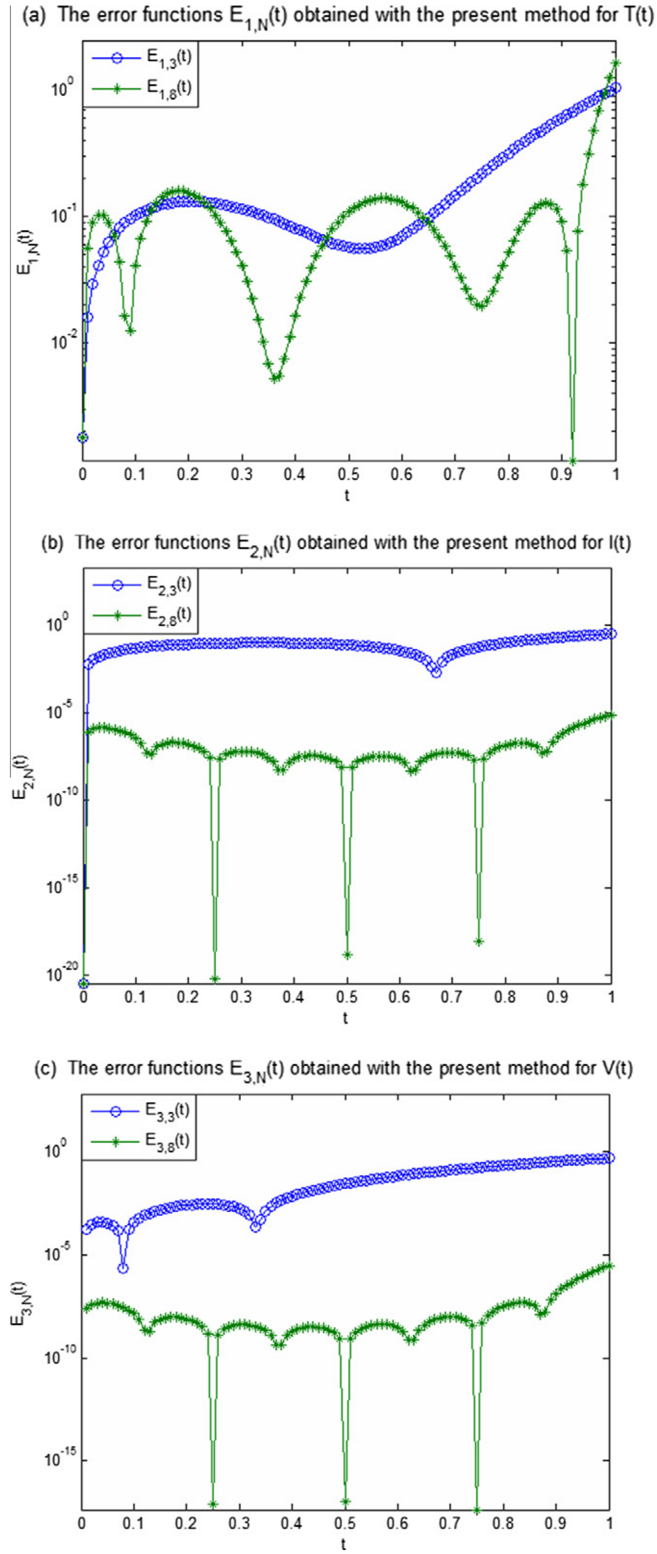

Fig. 2. For $N = 3, 8$ with the present method in the interval $0 \leqslant t \leqslant 1$, (**a**) graph of the error functions obtained with accuracy of solution $T_N(t)$, (**b**) graph of the error functions obtained with accuracy of solution $I_N(t)$ and (**c**) graph of the error functions obtained with accuracy of solution $V_N(t)$.

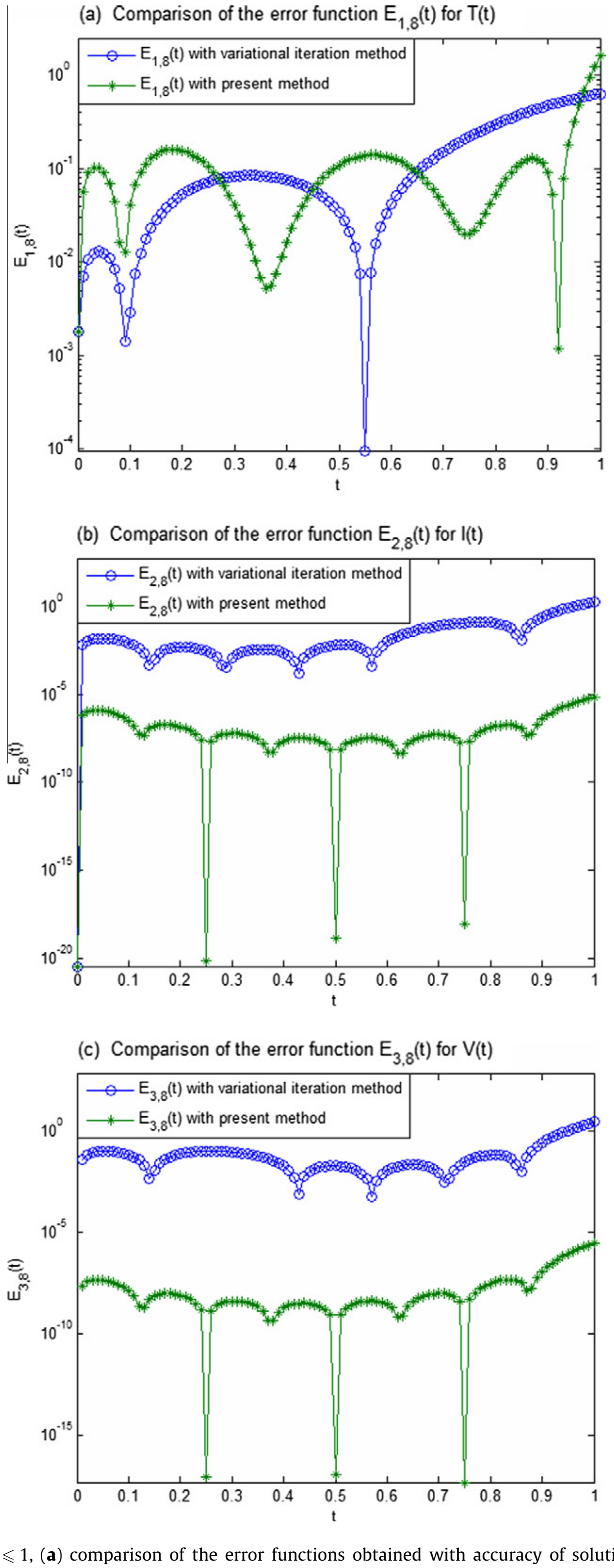

Fig. 3. For $N = 8$ in the interval $0 \leqslant t \leqslant 1$, (**a**) comparison of the error functions obtained with accuracy of solution $T_N(t)$, (**b**) comparison of the error functions obtained with accuracy of solution $I_N(t)$ and (**c**) comparison of the error functions obtained with accuracy of solution $V_N(t)$.

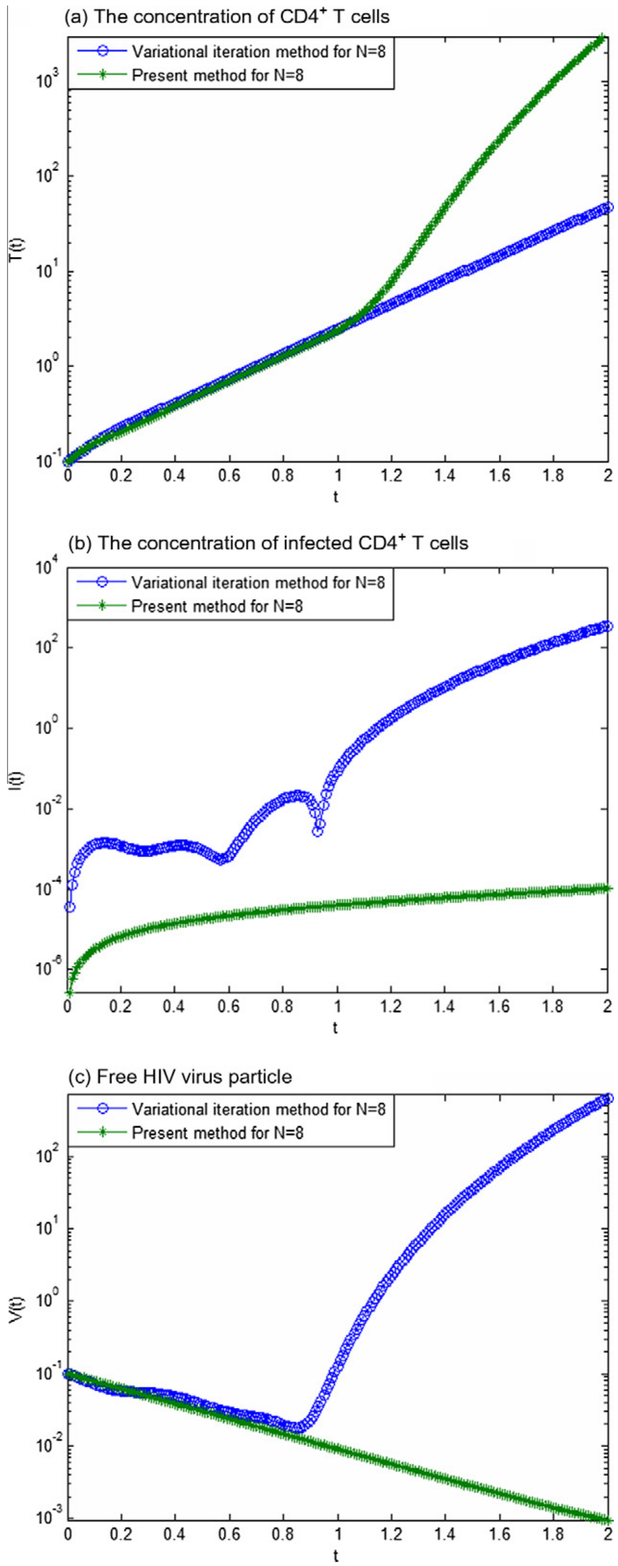

Fig. 4. For $N = 8$ in the interval $0 \leqslant t \leqslant 2$, (**a**) comparison of the approximate solutions $T_N(t)$, (**b**) comparison of the approximate solutions $I_N(t)$ and (**c**) comparison of the approximate solutions $V_N(t)$.

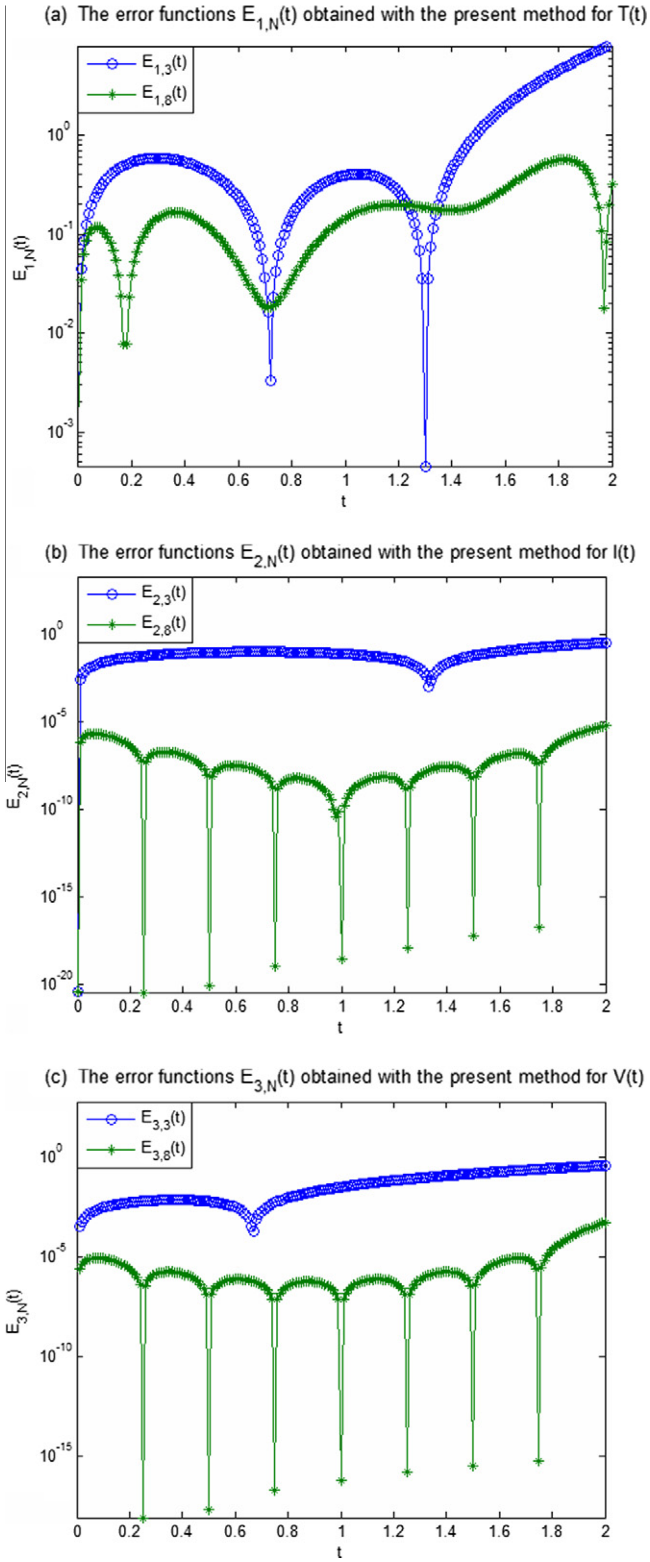

Fig. 5. For $N = 3, 8$ with the present method in the interval $0 \leqslant t \leqslant 2$, (**a**) graph of the error functions obtained with accuracy of solution $T_N(t)$, (**b**) graph of the error functions obtained with accuracy of solution $I_N(t)$ and (**c**) graph of the error functions obtained with accuracy of solution $V_N(t)$.

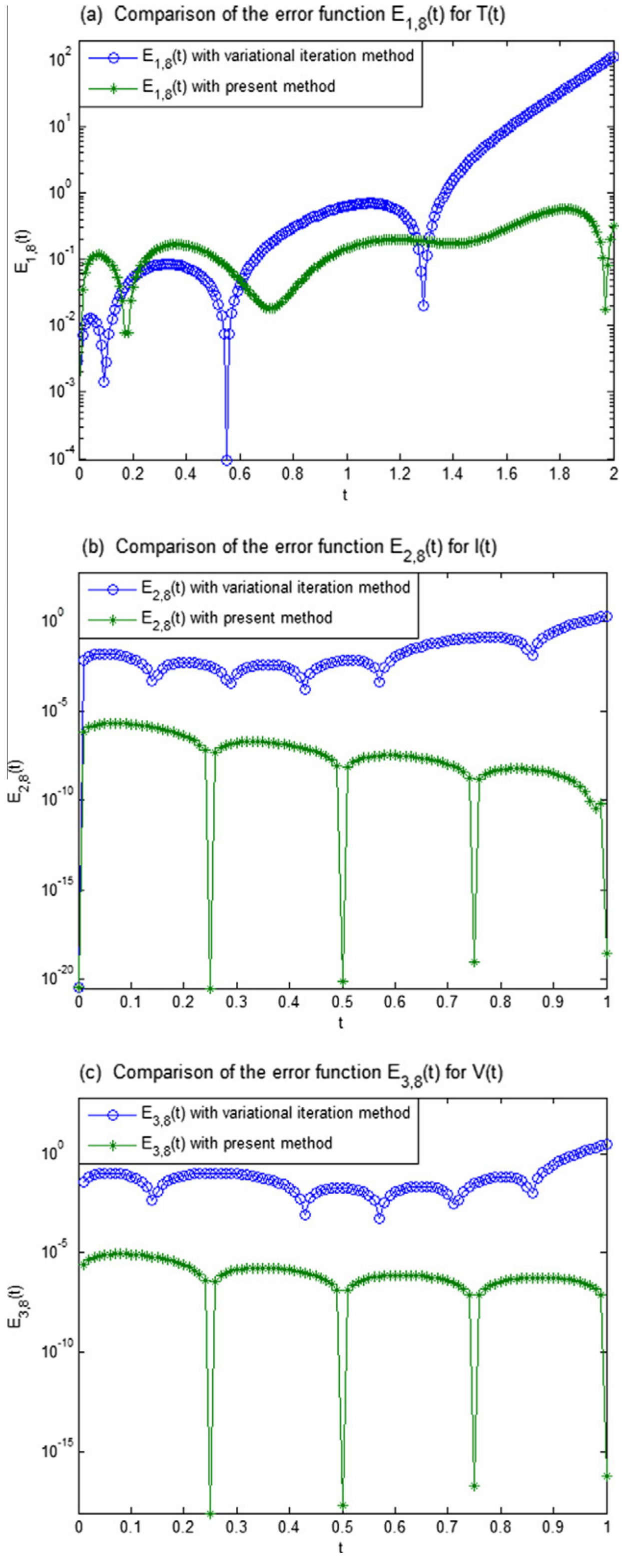

Fig. 6. For $N = 8$ in the interval $0 \leqslant t \leqslant 2$, (**a**) comparison of the error functions obtained with accuracy of solution $T_N(t)$, (**b**) comparison of the error functions obtained with accuracy of solution $I_N(t)$ and (**c**) comparison of the error functions obtained with accuracy of solution $V_N(t)$.

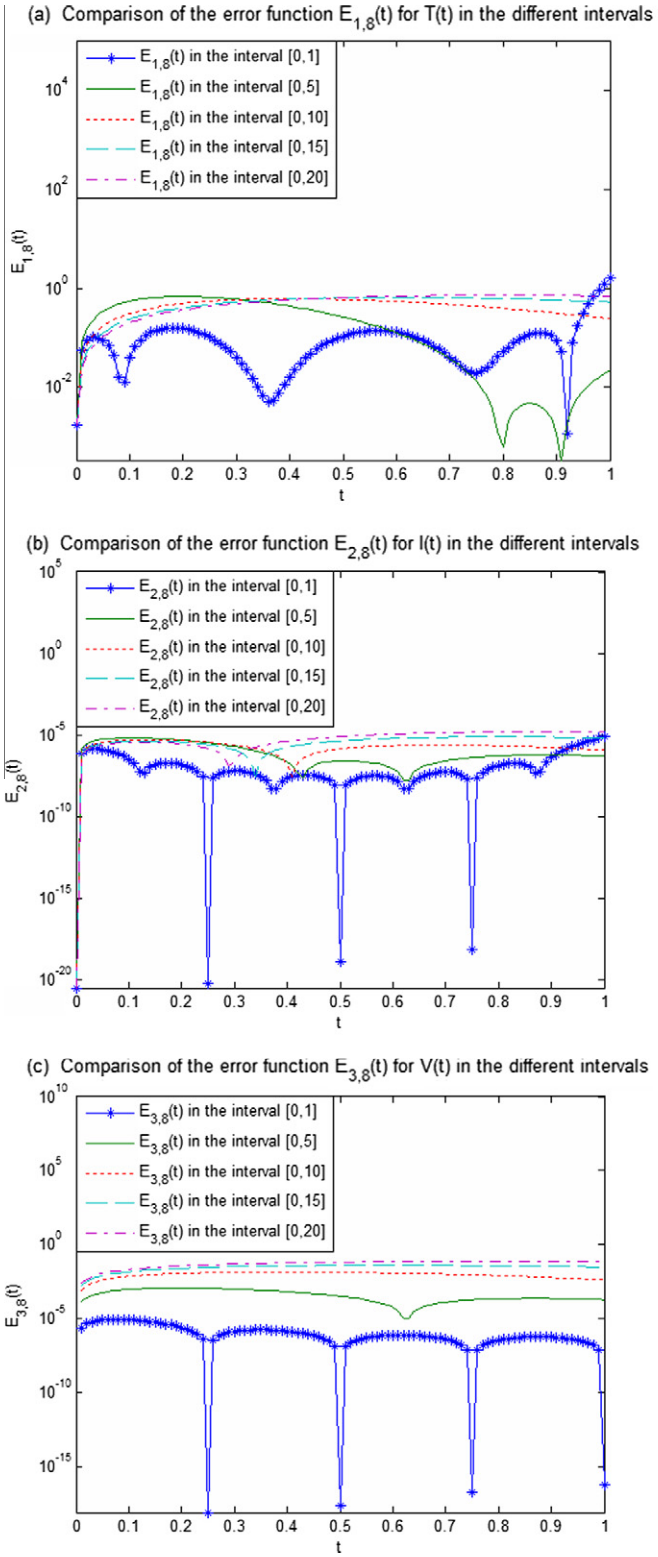

Fig. 7. For $N = 8$ in the different intervals, (**a**) comparison of the error functions obtained with accuracy of solution $T_N(t)$, (**b**) comparison of the error functions obtained with accuracy of solution $I_N(t)$ and (**c**) comparison of the error functions obtained with accuracy of solution $V_N(t)$.

Table 1
Numerical comparison for $T(t)$.

t_i	LADM–Pade [6]	Runge–kutta	MVIM [8]	VIM [8]	Present method for $N = 8$
0	0.1	0.1	0.1	0.1	0.1
0.2	0.2088072731	0.2088080833	0.2088080868	0.2088073214	0.2038616561
0.4	0.4061052625	0.4062405393	0.4062407949	0.4061346587	0.3803309335
0.6	0.7611467713	0.7644238890	0.7644287245	0.7624530350	0.6954623767
0.8	1.3773198590	1.4140468310	1.4140941730	1.3978805880	1.2759624442
1	2.3291697610	2.5915948020	0.2088080868	2.5067466690	2.3832277428

Table 2
Numerical comparison for $I(t)$.

t_i	LADM-Pade [6]	Runge–kutta	MVIM [8]	VIM [8]	Present method for $N = 8$
0	0	0	0.1e−13	0	0
0.2	0.603270728e−5	0.6032702150e−5	0.60327016510e−5	0.6032634366e−5	0.6247872100e−5
0.4	0.131591617e−4	0.1315834073e−4	0.13158301670e−4	0.1314878543e−4	0.1293552225e−4
0.6	0.212683688e−4	0.2122378506e−4	0.21223310013e−4	0.2101417193e−4	0.2035267183e−4
0.8	0.300691867e−4	0.3017741955e−4	0.30174509323e−4	0.2795130456e−4	0.2837302120e−4
1	0.398736542e−4	0.4003781468e−4	0.40025404050e−4	0.2431562317e−4	0.3690842367e−4

Table 3
Numerical comparison for $V(t)$.

t_i	LADM-Pade [6]	Runge–kutta	MVIM [8]	VIM [8]	Present method for $N = 8$
0	0.1	0.1	0.1	0.1	0.1
0.2	0.06187996025	0.06187984331	0.06187990876	0.06187995314	0.06187991856
0.4	0.03831324883	0.03829488788	0.03829595768	0.03830820126	0.03829493490
0.6	0.02439174349	0.02370455014	0.02371029480	0.02392029257	0.02370431860
0.8	0.009967218934	0.01468036377	0.01470041902	0.01621704553	0.01467956982
1	0.003305076447	0.009100845043	0.009157238735	0.01608418711	0.02370431861

$$\begin{cases} T_3(x) = 0.1 + 0.397953x - 0.139246852541x^2 + 1.53331521278x^3, \\ I_3(x) = 0.27e - 4x + 0.289995348988x^2 - 0.298999226352x^3, \\ V_3(x) = 0.1 - 0.24x + 0.298247385024x^2 - 0.542232141015e - 1x^3 \end{cases}$$

and

$$\begin{cases} T_3(x) = 0.1 + 0.397953x + 4.21749621198x^2 - 46.8374786567x^3 + 230.175269339x^4 - 558.172968834^{*}x^5 \\ \qquad +725.746949148x^6 - 482.182048971x^7 + 128.938056508x^8, \\ I_3(x) = 0.27e - 4x + (0.642886469976e - 4)x^2 - (0.412063752317e - 3)x^3 + (0.140158881191e - 2)x^4 \\ \qquad -(0.255342812657e - 2)x^5 + (0.255533881388e - 2)x^6 - (0.131574760141e - 2)x^7 + (0.269931631166e - 3)x^8, \\ V_3(x) = 0.1 - 0.24x + 0.288039040470x^2 - 0.230346760510x^3 + 0.137777316098x^4 - (0.648907032780e - 1)x^5 \\ \qquad +(0.239046107135e - 1)x^6 - (0.622770906496e - 2)x^7 + (0.843508602378e - 3)x^8. \end{cases}$$

For $N = 3, 8$ in interval $0 \leqslant t \leqslant 2$, we get the approximate solutions, respectively,

$$\begin{cases} T_3(x) = 0.1 + 0.397953x - 1.59009333124x^2 + 1.56477517051x^3, \\ I_3(x) = 0.27e - 4x - 0.740465439362e - 1x^3 + 0.139971313972x^2, \\ V_3(x) = 0.1 - 0.24x + 0.272037671859x^2 - 0.724045813030e - 1x^3 \end{cases}$$

and

$$\begin{cases} T_3(x) = 0.1 + 0.397953x + 2.62731836438x^2 - 11.6921722967x^3 + 29.1204428855x^4 - 33.9448986838x^5 \\ \qquad +22.1742503561x^6 - 7.35163621462x^7 + 1.06279788403x^8, \\ I_3(x) = 0.27e - 4x + (0.569107121175e - 4)x^2 - (0.174978901483e - 3)x^3 + (0.308935173003e - 3)x^4 \\ \qquad -(0.302863950946e - 3)x^5 + (0.169015756614e - 3)x^6 - (0.502317732093e - 4)x^7 + (0.618960122494e - 5)x^8, \\ V_3(x) = 0.1 - 0.24x + 0.287903160914x^2 - 0.229316016338x^3 + 0.134595618678x^4 - (0.596985128857e - 1)x^5 \\ \qquad +(0.191703501753e - 1)x^6 - (0.393310524827e - 2)x^7 + (0.379787858944e - 3)x^8. \end{cases}$$

In Fig. 1, the approximate solutions $T_N(t)$, $I_N(t)$ and $V_N(t)$ of the present metod applied for $N = 8$ in the interval $0 \leqslant t \leqslant 1$ are compared with the variational iteration method (VIM) [8] for $N = 8$. For the approximate solutions $T_N(t)$, $I_N(t)$ and $V_N(t)$ gained by the present metod for $N = 3, 8$ in the interval $0 \leqslant t \leqslant 1$, we denotes the error functions obtained the accuracy of the solution given by Eq. (23) in Fig. 2. For $N = 8$, the error functions obtained with accuracy of the solutions by using the present method are compared with the VIM in Fig. 3. For $N = 8$ in the interval $0 \leqslant t \leqslant 2$, the approximate solutions $T_N(t)$, $I_N(t)$ and $V_N(t)$ of the present metod are compared with the VIM in Fig. 4. Fig. 5 shows the error functions (23) gained by the suggested method for $N = 3, 8$ in the interval $0 \leqslant t \leqslant 2$. In Fig. 6, we give the comparisons of the error functions (23) with the current method and the VIM for $N = 8$ in the interval $0 \leqslant t \leqslant 2$. It is seen from Figs. 3 and 6 that the error functions gained by the present method is better than that obtained by the VIM. Thus, we say that the approximate solutions obtained by the present method is better than that obtained by the VIM. Fig. 7 displays the comparisons the error functions (23) with the present method for $N = 8$ in the different intervals. It is observed from Fig. 7 that the errors are some increase when the suggested method is applied for the same N by expanding the time interval. Therefore, the better results may be obtained by increasing value N when the time interval is expanded. The numerical values of the approximate solutions $T_N(t)$, $I_N(t)$ and $V_N(t)$ of the present metod for $N = 8$ in the interval $0 \leqslant t \leqslant 1$ are compared with the variational iteration method (VIM) [8], the modifield variational iteration method (MVIM) [8], the Laplace Adomian decomposition-pade method(LADM-pade) [6] and the Runge–kutta method in Tables 1–3. We can say that the numerical solutions of the current method is better than the other methods since the error function gained by the present method is better than that gained by the VIM in Figs. 3 and 6 and the numerical solutions of the VIM are quite close to the numerical solutions of the MVIM, LADM-pade and the Runge–kutta method in Tables 1–3. From Figs. 1 and 4, it is observed that, $T(t)$, the concentration of susceptible $CD4^{+}T$ cells increases speedily, $I(t)$, the number of $CD4^{+}T$ cells infected by the HIV viruses increases to a steady state of 0.07 for $N = 5, 8$ and $V(t)$, the number of free HIV virus particles in the blood decreases in a very short time after the onset of infection.

4. Conclusions

In this paper, the Bessel collocation method has been developed for finding approximate solutions of HIV infection model of $CD4^{+}T$ which a class of nonlinear ordinary differential equation systems. We have demonstrated the accuracy and efficiency of the present technique with an example. We have assured the correctness of the obtained solutions by putting them back into the original equation with the aid of Maple, it provides an extra measure for confidence of the results. Graphs of the error functions gained the accuracy of the solution show the effectiveness of the present scheme. It seems from Figs. 2 and 5 that the accuracy of the solutions increases as N is increased. The better results may be obtained by increasing value N when the time interval is expanded. This situation can be interpreted from Fig. 7. The comparisons of the present method by the other methods show that our method gives better results. Because it is observed from Figs. 3 and 6 that the error function obtained by the current method is better than that obtained by the VIM [8] and it is seen from Tables 1–3 that the numerical solutions of the VIM [8], the MVIM [8], LADM-pade [6] and the Runge–kutta method are almost same. A considerable advantage of the method is that the approximate solutions can be calculated easily in shorter time with the computer programs such as Matlab, Maple and Mathematica. The computations associated with the example have been performed on a computer by aid of a computer code written in Maple 9. The basic idea described in this paper is expected to be further employed to solve other similar nonlinear problems.

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