mathematics

Article

Path-Based Visibility Graph Kernel and Application for the Borsa Istanbul Stock Network

Ömer Akgüller [1], Mehmet Ali Balcı [1,*], Larissa M. Batrancea [2,*] and Lucian Gaban [3]

[1] Department of Mathematics, Muğla Sıtkı Koçman University, 48000 Muğla, Turkey
[2] Department of Business, Babeş-Bolyai University, 400174 Cluj-Napoca, Romania
[3] Faculty of Economics, "1 Decembrie 1918" University of Alba Iulia, 510009 Alba Iulia, Romania
* Correspondence: mehmetalibalci@mu.edu.tr (M.A.B.); larissa.batrancea@ubbcluj.ro (L.M.B.)

Abstract: Using networks to analyze time series has become increasingly popular in recent years. Univariate and multivariate time series can be mapped to networks in order to examine both local and global behaviors. Visibility graph-based time series analysis is proposed herein; in this approach, individual time series are mapped to visibility graphs that characterize relevant states. Companies listed on the emerging market index Borsa Istanbul 100 (BIST 100) had their market visibility graphs collected. To further account for the local extreme values of the underlying time series, we constructed a novel kernel function of the visibility graphs. Via the provided novel measure, sector-level and sector-to-sector analyses are conducted using the kernel function associated with this metric. To examine sectoral trends, the COVID-19 crisis period was included in the study's data set. The findings indicate that an effective strategy for analyzing financial time series has been devised.

Keywords: visibility graphs; stock market network; graph kernel

MSC: 05C82; 91G45; 91G50

Citation: Akgüller, Ö.; Balcı, M.A.; Batrancea, L.M.; Gaban, L. Path-Based Visibility Graph Kernel and Application for the Borsa Istanbul Stock Network. *Mathematics* **2023**, *11*, 1528. https://doi.org/10.3390/math11061528

Academic Editor: María del Carmen Valls Martínez

Received: 17 February 2023
Revised: 10 March 2023
Accepted: 13 March 2023
Published: 21 March 2023

1. Introduction

Recent years have seen the development of graph network approaches to the analysis of time series. In these methods, a time series is mapped into a graph network, and the features of the time series are considered to be inherited in the mapped network. The mapped network can then be examined from the perspective of a complex network. The visibility graph network method, for example, has been shown to indicate the fractal characteristic of a time series by the power law degree distribution and the small world path length of the corresponding network [1]. This was accomplished by using the small world path length feature of the network. This visibility graph method is an effective approach that has been used to successfully characterize human strive intervals [2], hurricane occurrence in the United States [3], foreign exchange rates [4], energy dissipation rates in fully developed three-dimensional turbulence [5], human heartbeat dynamics [6], electroencephalogram series [7], and stock indices [8,9].

Further analysis of the time series and its underlying system is facilitated by applying measures of complex network structure to the newly built network. Under the umbrella of this overarching framework, a wide variety of methodologies, such as cycle networks [10], correlation networks [11], recurrence networks [12–14], transition networks [15], visibility graphs [16], and many others, have been developed and presented.

Visibility graphs represent a mathematical tool that has been widely used in various fields to analyze complex systems and understand their behavior. In the realm of finance, visibility graphs have gained increasing attention in recent years as a tool for analyzing and predicting the movements of financial assets such as stocks and bonds.

Recently, some fascinating methods for transforming time series into networks have been proposed. These algorithms enable researchers to learn more about time series by investigating the attributes of the network they create. In recent years, a variety of different strategies for converting time series into complicated network-like mappings have been developed. In most cases, these methods are separated into three distinct types. The first is known as the visibility graph, and it is predicated on the investigation of the convexity of successive values in a time series [17]. In the end, the purpose of these methods is to build a bridge between the nonlinear analysis of time series, dynamic systems, and the theory of graphs by accurately recreating information found in time series using an alternative mathematical framework.

Network science has developed into a powerful tool for analyzing complex systems such as financial markets. It allows one to study the properties and dynamics of financial systems represented as networks of interconnected entities, such as stocks, companies, investors, financial institutions. In [18], authors study the impact of network complexity on financial stability and systemic risk. The research demonstrates that the structure of financial networks can amplify shocks and contagion, resulting in market disruptions on a large scale. In addition, the authors propose a framework for measuring systemic risk based on network topology and apply it to an analysis of the interbank lending network in Europe. In [19], researchers use network analysis to investigate the correlations between stock market benchmarks in various nations. To determine which groups of nations are highly interdependent, they build a network of cross-correlations between market indices. They also demonstrate the dynamic nature of the network's most important metrics by means of centrality measures. The structural significance of financial networks is the basis for a novel valuation methodology proposed in [20]. To quantify how much a component increases the network's overall vulnerability, the authors devise a metric of centrality. Using the European interbank market as an example, they demonstrate that the metric successfully identifies the most critically significant institutions.

In addition to the characteristics of globally dominant markets, network science studies are extremely useful for the analysis of emerging markets, particularly in the context of systematic risk. For instance, [21] shows that lowering systemic risk from the three risk sources can be accomplished by raising the average degree of companies in the bank-firm credit network, the density of the bank-asset portfolio network, and the bank capital sufficiency ratio. Furthermore, a bank's exposure to the dangers of a specific market increases with the proportion of bank assets that are invested there. One study [22] used a high dimensional financial network to examine the spread of systemic risk across China's various sectors, as well as the effects of monetary policy and the diversity of the country's businesses. According to empirical data, the global financial crisis of 2008 and the 2015–2016 stock market crash contributed both to a substantial rise in the overall degree of systemic risk. Statistics from [23] demonstrate that the Tehran Stock Exchange's cross-shareholding network is, in fact, a scale-free network. The stock market is ripe for a catastrophe in the event of systemic risk, as evidenced by the presence of scale-free networks. One study [24] suggests a technique for examining the effect of causality in a statistically built network of correlations between financial indices. The results show the significance of correctly recognizing and utilizing finance networks in the problem-prediction process. In [25], authors discuss the major developments in the literature on network robustness and finance networks, providing a holistic view of the topic. Financial network intelligence, network resilience measures, financial regulatory technologies, and regulatory applications are also tackled along with the uses of financial network resilience processing in financial regulation. Numerous studies [26–33] investigated the significance of systematic risk techniques for Borsa Istanbul and the structural change in the market during various stress periods. These studies can be found within the vast body of research that has been cited previously.

The construction of a visibility graph involves mapping the time series data of a financial asset onto a graph, with each data point represented by a vertex. The edges of the graph are then determined based on the visibility relationship between data points. This relationship is defined by an edge that is drawn between two vertices if the line connecting them is not intersected by any other data points. The resulting graph can be used to analyze the underlying patterns and trends in the financial asset's data, and it can provide valuable insights for investors and traders.

Since visibility graphs provide a straightforward geometric interpretation of the primary time series, they are one of the methods that lend themselves particularly well to quantitative research into the evolution of market segmentation. Time series are a sequence of characteristics that can be used as a valuable indicator of the structural and dynamic nature of most complex systems and have been collected over a period of time.

One of the key advantages of visibility graphs is their ability to capture both long-range and short-range correlations in the data. This is particularly useful in the context of stock markets, where the behavior of financial assets is often influenced by a wide range of factors, including economic indicators, political events, and market sentiment. By identifying these correlations, visibility graphs can help investors and traders in making informed decisions about which assets to buy or sell, and when to perform these tasks.

In addition to their use in analyzing individual financial assets, visibility graphs can be used to identify correlations between different assets. This can be particularly useful for diversifying portfolios and mitigating risks. For example, if two assets are strongly correlated, an investor may choose to reduce their exposure to one of the assets in order to balance their portfolio. On the other hand, if two assets are uncorrelated, an investor may choose to increase their exposure to both assets, diversify their portfolio and reduce overall risk. The application of visibility graphs in stock markets has the potential to provide valuable insights and assist in making informed decisions in the complex and dynamic world of finance. Therefore, this is an area of active research and development, expected to register continuous progress and advancement in the coming years.

In this study, visibility graphs such as the abovementioned effective utilization technique, are employed to analyze a stock market influenced by the COVID-19 pandemic. Our strategy is predicated on the definition of the kernel function, a measure of the similarity between two graphs. This novel kernel function is created using the Wasserstein distance function of the distributions arising from a vertex's inclusion in the visibility graph of three paths. While three paths provide a fairly compact study region in big networks, they also account for the local maximum values when examining the time series from which the visibility graph is derived. Specifically, it compares by capturing the abrupt local changes in the time series' values. This comparison addresses the immediate consequences of economic stress on financial time series. The kernel function is used to conduct a cluster analysis, where firms listed on the BIST 100 index are evaluated within a sectoral framework.

To the best of our knowledge, this study presents for the first time a kernel method, specified on the visibility graphs, that proves to be a powerful instrument for time series analysis. We introduce a novel approach to comparing time series over their intricate interrelationships, complementing the topological metrics of network theory. We believe that this technique, which we tested on the Borsa Istanbul during a time of market duress, has broad utility.

2. Methodology

2.1. Data Set

Borsa Istanbul 100 Index (BIST 100) is an index of the top 100 businesses listed on Borsa Istanbul, the principal stock exchange in Turkey. The index captures the overall performance of the Turkish stock market, and it is one of the most watched indexes in the nation.

The BIST 100 index is computed using a free float-adjusted market capitalization weighting approach, which implies that the weight of each component stock is based on its market capitalization relative to the total market capitalization of all companies in the index. The index is adjusted for the free float of each component stock, which means the weight of a stock is based on the shares that are available for trading as opposed to the total number of outstanding shares.

The BIST 100 index covers financial, industrial, consumer products, and technology businesses, among others. More than half of the index is comprised of the financial, industrial, and consumer goods sectors. The financial sector consists of banks, insurance firms, and other financial organizations, while the industrial sector comprises businesses engaged in manufacturing, construction, and other industrial operations. The consumer goods industry consists of businesses that manufacture and sell items, such as apparel, electronics, and home goods.

In general, the BIST 100 index is a valuable instrument for investors and analysts seeking to comprehend the performance of the Turkish stock market. The index gives a wide and representative view of the market as a whole by monitoring the performance of the top 100 firms listed on Borsa Istanbul.

COVID-19, also known as the new coronavirus, has had a substantial effect on the Borsa Istanbul, the main stock market in Turkey. Due to the economic effect of the pandemic, Borsa Istanbul has witnessed severe volatility and reductions in value, as have many other stock markets throughout the globe. During the early stages of the pandemic, the BIST 100 index had a significant decrease in value as investors were more worried about the virus's economic effect [34–38]. This loss was worsened by a worldwide reduction in oil prices, which has had a substantial effect on the Turkish economy. As the epidemic has advanced, however, the BIST 100 index has somewhat rebounded as investors gained confidence in the Turkish economy's capacity to withstand the economic slowdown caused by the pandemic. The Turkish government has adopted monetary and fiscal policy measures to stabilize the market and assist the economy. Despite this comeback, COVID-19 has had a huge influence on Borsa Istanbul. Because of the pandemic, several firms listed on the market have faced revenue and profit reductions, while others have been forced to halt operations or declare bankruptcy. This has had a negative effect on the value of their equities, which has impacted on the BIST 100 index's overall performance. COVID-19 has influenced heavily Borsa Istanbul, since the stock market has registered tremendous volatility and decreases in value. Even if the market has somewhat recovered, the pandemic's long-term effect on the Turkish economy and the Borsa Istanbul remains visible [39,40].

In this study, a total of 17 sectors with corresponding sub-sectors are considered in order to present more detailed characteristics. These sectors were: Banks, Simple Metals, Information Technologies, Electricity, Financials, Real Estate Investment, Service, Holding, Chemicals, Leasing and Factoring, Insurance, Technology, Textile, Telecommunication, Tourism, Transport, Food and Beverage. To investigate the effects of the COVID-19 pandemic on the BIST 100 index in the sectoral context, the beginning of the period of analysis was set to March 2020, the period when COVID-19 was first detected in Turkey, and the end was set to March 2022. Hence, we examined a two-year period. The time series of each sector are depicted in Figure A1 and the descriptive statistics are presented in Table A1.

2.2. Visibility Graphs

A sequence of data points obtained from regularly spaced points over time and listed in time order is called a time series. Specifically, a time series is any data series obtained in discrete time. Data can be obtained from real-life measurements or from a mathematical model. A graph (or network) is a mathematical structure consisting of a collection of objects to which some pairs of objects are connected or related in a certain sense. A graph G is generally represented by the pair $G = (V, E)$. Here, V represents the set of objects, which form the vertices of the graph. In the case of a connection or a relationship between objects, an edge is drawn between the two vertices. The set of these edges is also denoted by E. These nodes are called adjacent since the objects that are related to each other are joined by an edge in the graph. Mathematically, it is defined as $E \subset V \times V$.

Adjacency matrices are very important in the representation of graphs. This is a square matrix of 1s or 0s. The adjacency matrix of any graph with n vertices, $A = \left[a_{ij}\right]$ is a matrix of $n \times n$ type, and its inputs are defined as follows:

$$a_{ij} = \begin{cases} 1, & (i,j) \in E \\ 0, & otherwise \end{cases}. \tag{1}$$

The relationship between the nodes in the graphs can be direct or indirect. If there is a direct relationship between two nodes, these nodes are joined by an edge. In some cases, an indirect relationship can be established over different vertices. If at least one communication can be found between any two pairs of vertices in a graph, this graph is called a connected graph, and if there is no communication for even at least one pair of vertices, the graph is called a disconnected graph.

Graphs can also be classified as directed or undirected. In directed graphs, edges have initial and terminal vertices, and the relationship is unidirectional. Therefore, when $(u,v) \in E$, $(v,u) \notin E$. There is no concept of direction in undirected graphs, the relations are symmetric, namely both (u,v) and (v,u) are members of the edge set. In some cases, numerical values, such as distance, cost, and length can be assigned to the graph edges. Graphs where each edge has a numerical weight are called weighted graphs. On the other hand, an unweighted graph has no associated weight information, and in this case, the edges are considered as equally weighted. A path in a graph is an ordered set of edges that connects some collection of vertices. A path's beginning and ending vertices need not be adjacent on the graph, and it is possible for the path to loop back on itself. A path's length is equal to the sum of its edge counts. The shortest paths between any two nodes are those that use the fewest possible edges and its length gives the graph distance between the starting and the ending vertices. If the path's beginning and ending vertices coincide, this is called a cycle. A tree is a special case of an undirected, connected, and cycle-free graph.

Time series analysis of financial processes is very effective in the correct interpretation and detailed analysis of financial data. With this analysis, the behavior of the data can be understood and predictions can be made for the next processes. Many methods have been developed within the scope of time series analysis, which has been popular for a long time [41–46]. Despite all this progress, many time series analysis methods are subject to some prerequisites for data. One of these conditions is stationarity. When the time series is stationary, a meaningful analysis cannot be obtained for the financial time series, since it will have a fixed mean and variance. In this case, the data should be transformed according to the method used in the analysis of financial time series. Therefore, new methods that can be used in the analysis of financial time series without the need for these transformations were needed and researchers turned to the complex network theory [47,48].

In this study, time series data are analyzed using the visibility graph method, which entails constructing a graph based on the visibility relationships between data points. In contrast, the theoretical methods of the conventional graph entail constructing a graph based on the direct connections between data points. When compared to other graph theory techniques for modeling finance time series, the visibility graph approach has several advantages. First, financial time series data frequently display nonlinear relationships among data points. The visibility graph method is superior to direct connections for capturing these non-linear relationships, as it incorporates the visibility relationships between data elements. Moreover, financial time series data can be sparse, with several data points lacking. The visibility graph method can deal with absent data points by building a graph based on the visibility relationships between available data points. Furthermore, the fundamental trends in financial time series data may be difficult to discern due to the presence of noise. Through the view graph's creation, the technique can level the data and lessen the impact of disturbance. Finally, with its low barrier to entry and lack of need for specialist tools, the visibility graph technique is a workable option for the analysis of finance time series data. When compared to more traditional graph theory techniques, the visibility graph approach stands out as a versatile and effective method for studying financial time series data.

A graphical depiction of a predetermined set of points in a plane is known as a visibility graph. It is formed by linking pairs of points that are able to “see” each other, which means that there is a straight-line segment between them that is unobstructed by any other points that may be in the way of their vision of each other. Visibility graphs have the capability of representing the topological structure of a collection of points, which is one of their many desirable properties. Visibility graphs can also depict the connection of a group of points, which is another of their many useful properties.

To analyze the time series effectively and in detail, it is necessary to consider the time series as a network. It is normal to experience a certain amount of information loss depending on the methods to be used to treat a time series as a network. Similar to the classical linear time series analysis, some information may be lost in network analysis. However, depending on the algorithm to be used to model the time series as a network, it is possible to preserve the nonlinear properties of the series at the maximum level. This reveals the importance of the network model to be used. This is why visibility graphs, which are suitable for almost all time series, are frequently preferred in time series analysis. We first obtain the visibility graphs of the time series with the algorithm proposed by Lacasa et al. [1].

When data treated as time series are defined as $\{y_i = y(t_i)\}_{i=1}^{N}$, each y_i value corresponds to a data point in the form of (t_i, y_i), and each data point determines a top of the graph according to the proposed algorithm. If two vertices in the visibility graph can see each other, they are joined by an edge. For any two vertices having planar coordinates (t_i, y_i) and $\left(t_j, y_j\right)$, the ability of two vertices to see each other mathematically is defined as follows:

$$\frac{y_j - y_n}{t_j - t_n} > \frac{y_j - y_i}{t_j - t_i}, \ \ \forall t_n \in \left(t_i, t_j\right). \tag{2}$$

With the determination of the edges according to this relation, the visibility graph of the time series is obtained. Visibility graphs are connected and undirected graphs. With the visibility graph connected, it can be said that all nodes have at least one adjacent node. In the case of two nodes seeing each other, the direction will not be indicated on the edges, since there is an edge between them and the two nodes will see each other at the same time. As a result, the visibility graph is undirected. Moreover, the visibility graphs are invariant under the affine transformation of the time series. Specifically, the visibility graphs of the time series can be rescaled horizontally or vertically [1]. In Figure 1, we present a time series with 25 time-ticks and an emerging visibility graph as an example.

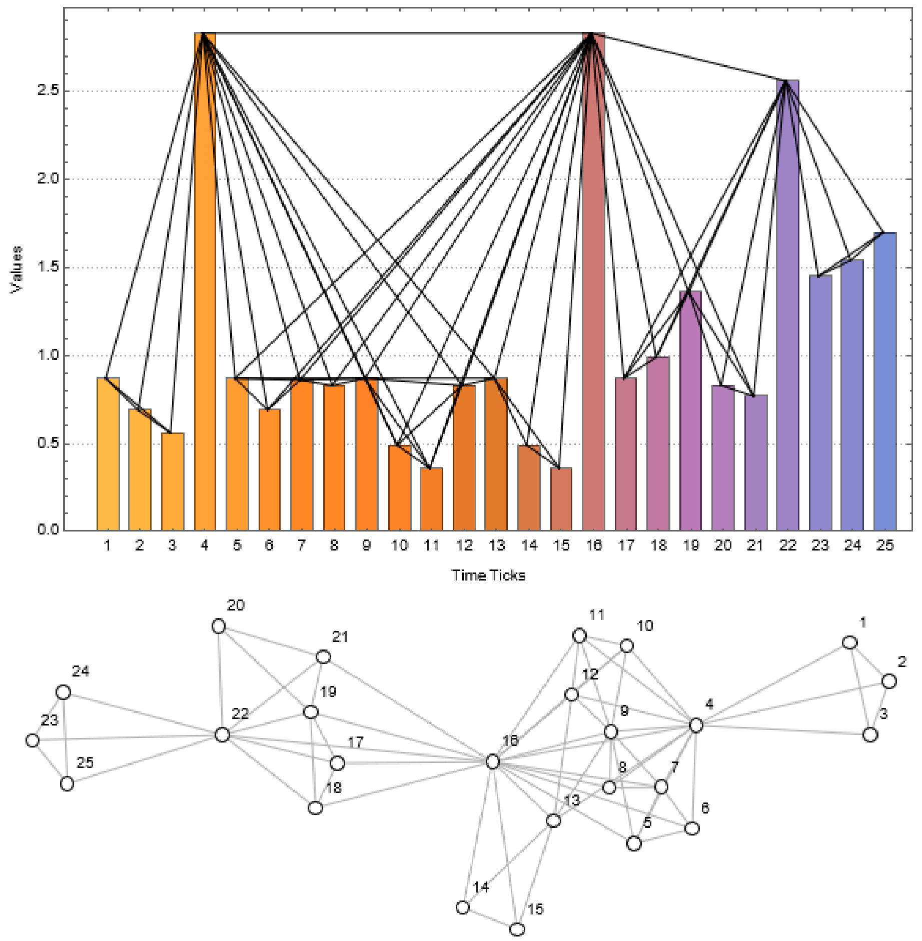

Figure 1. A time series (**upper sub-figure**) and emerging visibility graph (**lower sub-figure**).

2.3. Kernel Function

A function that determines the degree to which two graphs are similar is known as a graph kernel. In place of only contrasting the distinct nodes and edges, it offers a method for computing the degree to which the underlying structures of the graphs are analogous to one another. Graph kernels must be symmetric and positive semi-definite. By defining a kernel with the nodes of the graph, it can be decided how similar the two nodes are, or a kernel can be defined between graphs to compare two graphs in terms of similarity. The important thing is to be able to define a kernel that captures the natural characters of the graphs. In this study, we want to examine the visibility of graphs. Therefore, we will focus on the graph kernels used to measure graph similarity. We first introduce the novel vertex centrality measure developed in this study, and then we introduce the associated graph kernel function.

Recall that the path in a graph is defined as an ordered set of edges connecting some set of vertices. Let us denote a path graph with n vertices using P_n. The centrality measure presented in this study takes into account P_3. Specifically, a measure of vertices connecting second-order vertices that do not directly interact with each other is presented and denoted by P_3. In the context of visibility graphs, this is a measure of the activity of the intermediate trade day, which controls the interaction between trade days.

When calculating the P_3 centrality of a graph, the number of different P_3 paths connecting any two non-adjacent vertices on the graph is determined. The resulting number is the C_{P_3} centrality and is formally given by

$$C_{P_3} = \left|\left\{ i \in V \ : i \neq j \neq k, \quad a_{ij} = a_{ik} = 1 \; and \; a_{jk} = 0 \right\}\right|, \tag{3}$$

Since it is a measure of how many different pairs of vertices one vertex mediates where there is no direct communication, a high degree of centrality of C_{P_3} means that this vertex is an important part of the relationship it represents.

The C_{P_3} centrality distribution obtained for all vertices of the graph determines the heterogeneity of the graph in terms of vertex significance. The closeness of the values in this distribution enables inferences about the intensity of the relationship. The Wasserstein-1 distance function can be used to define a kernel function since this degree of importance is determined by the relationships between the graph's subgraphs.

Let the two probability densities be ρ_i and ρ_j in $\mathbb{R}^m$, and Π be the coupling between these densities. Then,

$$W_1(\rho_i, \rho_j) = \inf_{\pi \in \Pi(\rho_0, \rho_1)} \int_{\pi \in \Pi(\rho_0, \rho_1)} \| x - y \| \, \pi(dx, dy) \tag{4}$$

is called Wasserstein-1 distance. Moreover, by using Equation (4), we are able to define the kernel function as follows:

$$\kappa_{P_3}(G_1, G_2) = \frac{\lambda}{W_1(\rho_1, \rho_2)}, \tag{5}$$

where ρ_1 and ρ_2 are the probability densities of P_3, G_1 and G_2 are the centrality distributions of graphs, and λ is the normalization term. We shall note that κ_{P_3} is a symmetric and positive definite.

3. Results

This study looks at Borsa Istanbul's key sector trends from two perspectives. The first method examines the topological qualities of the visibility graphs collected for a specific subperiod to see how the dynamics of each sector have changed over time. The second method uses the kernel function to determine similarities between visibility graphs during two years on data acquired from different industries, as well as the dynamics of the sectors' interactions with one another. Clusters in distinct industries can be constructed using the matrices produced by the kernel functions.

3.1. Intra-Sectorial Results

Each of the time series that were obtained for the study included a total of 501 individual entries. To examine the trading process over a span of 2 weeks as a series of subperiods, sliding windows with ten entries and one offset from each time series are utilized to produce time series families of sectors. As a result, 492 families of time series are created, each with ten-time entries belonging to a different sector.

We obtained graph structures from the time series of subperiods using the method of generating the visibility graph, which is detailed the method section. The topological changes in these graph structures are examined with the changes in average degree, average betweenness, average P_3 centrality, and global efficiency values.

The betweenness centrality of a vertex in graph theory is a metric for determining how heavily it influences the shortest pathways between other vertices. It quantifies how significant a given vertex is in a graph. Betweenness centrality in a graph is the proportion of all pairs of vertices that pass through a node, scaled by the total number of nodes with

$$C_B(i) = \sum_{i \neq j \neq k} \frac{\sigma_{jk}(i)}{\sigma_{jk}}, \tag{6}$$

where the sum of the shortest paths from vertices j and k is denoted by σ_{jk}, while the subset that includes the vertex i is denoted by $\sigma_{jk}(i)$. Since nodes on the shortest paths between other nodes exert the most control over the transmission and flow of information in a network, betweenness centrality is frequently used to determine which nodes are the most crucial.

The global efficiency of a network is a concept from graph theory that quantifies how effectively the graph's vertices are able to exchange information with one another. One way to characterize it is using the mean inverse shortest path distance from any two vertices in a network. The degree to which vertices are able to communicate with one another is quantified by the inverse of the shortest path length, with a larger number implying simpler communication. Graph global efficiency is determined by averaging the inverse shortest path lengths between every pair of vertices in the graph with

$$E_{gl}(i) = \frac{1}{|V|(|V|-1)} \sum_{i \neq j} \frac{1}{d_G(i,j)}, \tag{7}$$

where d_G is the graph distance function. The vertices of a network with a high global efficiency are well-connected and can easily interchange data with those of a graph with a lower global efficiency. To compare how effectively communication is allowed or to identify not communicable regions in a graph, one may accomplish both by analyzing the graph's global efficiency.

We shall note that communication in the context of financial networks is the sharing of data and the execution of financial transactions among various entities in the financial sector. This can involve the transfer of funds between institutions, as well as the sharing of financial information, such as pricing and trade volumes.

In Figures A2–A18, we present resulting calculations for intra-sectorial analysis. In this study, in which the concept of C_{P_3} centrality is used for the first time, the heterogeneity levels of the changes in the averages of the results are given in Table 1 using Shannon entropies. Moreover, the distributions of average C_{P_3} values throughout the sliding windows are presented in Figure 2 with box plots.

Table 1. Shannon entropies of intra-sectorial results.

Sector	Average Degree	Average Betweenness	Average C_{P3}	Global Efficiency
Banks	2.73252	5.27751	5.83803	4.79955
Simple Metals	2.68426	5.24899	5.80198	4.66781
Information Technologies	2.83218	5.34155	5.85802	4.8448
Electricity	2.7344	5.28616	5.83368	4.79945
Financials	2.75841	5.28165	5.86131	4.76096
Real Estate Investment	2.75179	5.34874	5.94941	4.80714
Service	2.67371	5.33939	5.86551	4.73453
Holding	2.74399	5.34092	5.9258	4.76948
Chemicals	2.74949	5.33542	5.88588	4.85055
Leasing and Factoring	2.78981	5.29507	5.83368	4.77155
Insurance	2.79066	5.25911	5.79617	4.86001
Technology	2.68044	5.18885	5.82957	4.69456
Textile	2.82487	5.38511	5.83059	4.83034
Telecommunication	2.64946	5.21591	5.81865	4.72057
Tourism	2.77732	5.40047	5.87713	4.76487
Transport	2.80226	5.38155	5.86698	4.80843
Food and Beverage	2.78738	5.33615	5.85031	4.87739

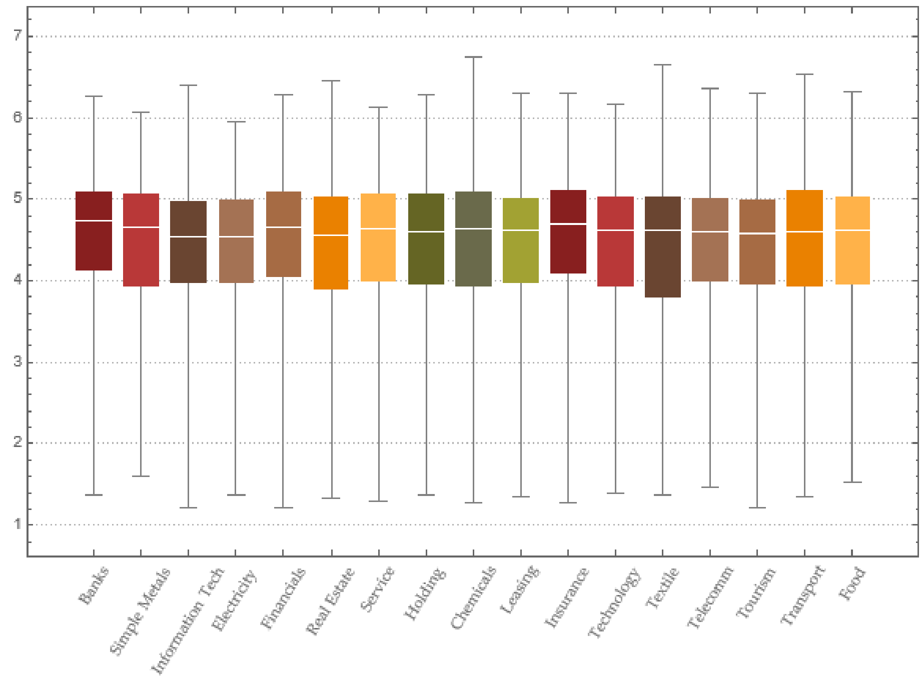

Figure 2. Distributions of average C_{P_3} values throughout the sliding windows.

3.2. Inter-Sectorial Results

Inter-sectorial relations are the connections that exist between the various markets that make up a stock exchange, and the term "relations" is used here. These markets can be categorized in a number of different ways, such as by the industry they belong to (such as technology, healthcare, or financial services), by the economic activity they participate in (such as consumer goods, industrial goods, or utilities), or by their total market capitalization (e.g., large cap, mid cap, small cap).

A stock market's overall performance may be affected by a wide range of variables, such as the state of the economy, advances in technology, and changes in regulatory policies, amongst others. For instance, if the economy is expanding, some industries, such as consumer goods and industrials, may have strong performance, whilst others, such as utilities, may have weak performance. In a similar vein, if there are substantial technical improvements made in a specific area, such as the discovery of a new medication or the release of a new product, then the sector that corresponds to that industry may outperform others.

When making judgments about their investments, investors could take into account the inter-sectoral interactions that exist within a stock market. For instance, an investor who is bullish on the technology sector may choose to allot a larger portion of their portfolio to technology stocks, while an investor who is bearish on the energy sector may choose to reduce their exposure to energy stocks. Both of these investors' decisions would be based on their own assessments of the respective markets. When putting up a diversified investment portfolio, the inter-sectorial relations of a stock market may be a valuable resource for gaining insight into the relative performance of various market segments and can be an essential consideration to take into account.

Taking into consideration the COVID-19 stress period, this study establishes a connection between the sectors in the BIST 100 index within the context of the kernel function determined by the new metric offered for visibility graphs. The visibility graphs derived from the 2-year time series were used to evaluate these relations. The degree, betweenness, and P_3 distributions of the resulting visibility graphs are shown in Figures A19–A35.

After getting the C_{P_3} distributions of the visibility graphs that are associated with 17 different sectors, the kernel function described in Equation (6) is used to conduct an assessment of the degree to which these 17 different sectors are comparable to one another throughout the graphs. The matrix of the corresponding kernel function is given in Figure 3.

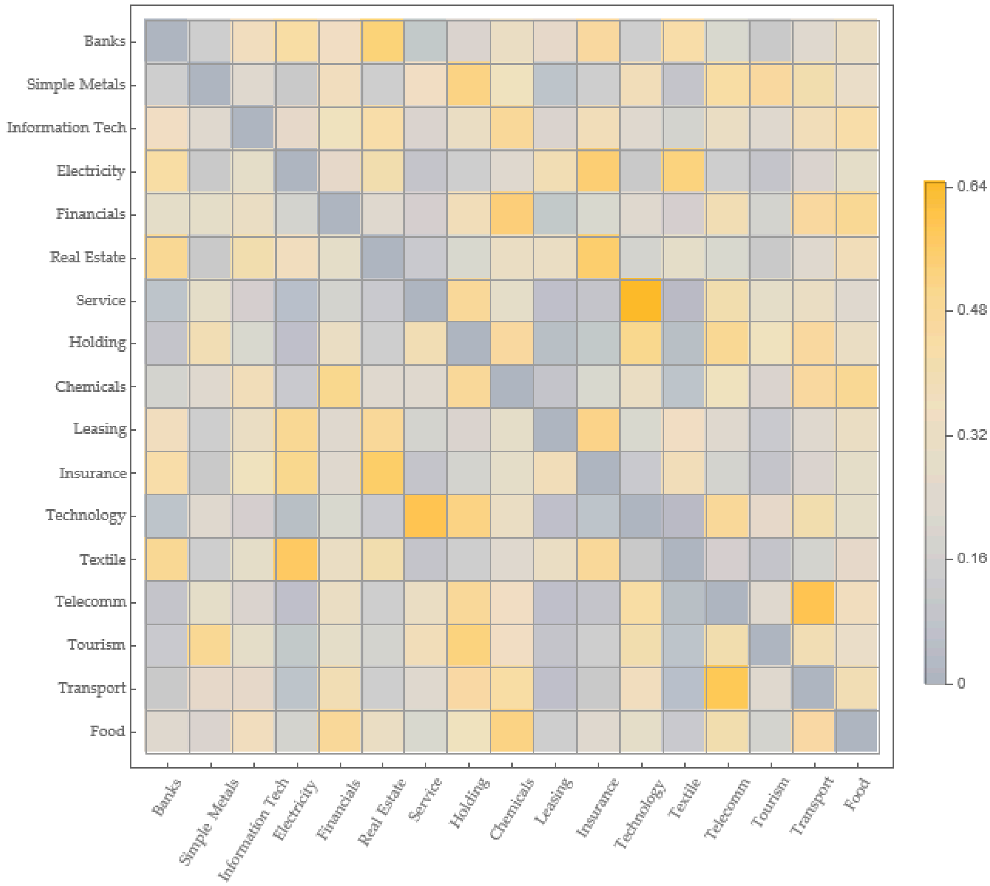

Figure 3. Kernel matrix for visibility graph similarity.

It is possible to identify clusters of sectors using the similarity matrix obtained via the kernel function. In this study, the hierarchical clustering method was used as a clustering approach. The resulting hierarchical tree is presented in Figure 4 and the clusters in Table 2.

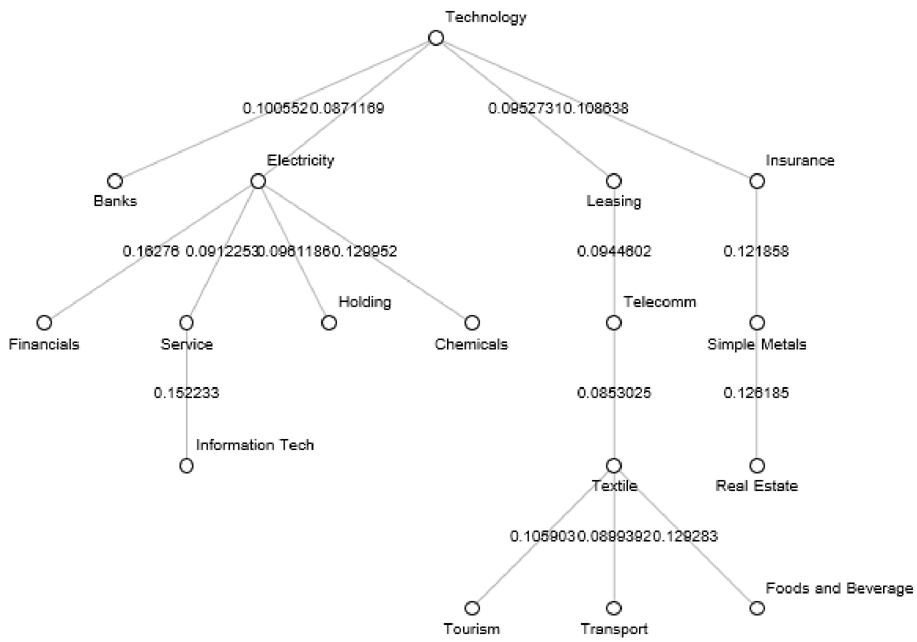

Figure 4. Clustering tree using kernel matrix.

Table 2. Sectorial clusters.

Cluster No.	Sectors			
1	Banks	Leasing and Factoring	Technology	Telecommunication
2	Electricity	Financials	Holding	Chemicals
3	Textile	Tourism	Transport	Foods and Beverages
4	Simple Metals	Real Estate Investment	Insurance	
5	Information Technologies	Service		

4. Discussion and Conclusions

Visibility graphs are a helpful tool for studying the connections between various financial instruments traded on the stock market as well as for spotting trends and patterns in the movement of stock prices. In this study, the sectoral assessments of the firms that are included in the BIST 100 index were performed using a newly constructed measure that was applied to the visibility graphs.

In the first step of our study, the changes in C_{P_3} centrality measures over time are examined and compared to other network topology metrics. These changes help to identify the second-order interactions of trading days that do not interact with each other. We observed that the average P3 values had the greatest amount of variability in relation to all the other industries. When the heterogeneity of these average changes is stated, we noticed that the average betweenness, global efficiency, and average degree values followed the C_{P_3} value. This is due to the fact that C_{P_3} is the value that best describes the distribution of these average changes. This demonstrates that the C_{P_3} centrality metric is applicable in a variety of situations, particularly those involving the stock market. The distribution of C_{P_3} values along the sliding windows is comparable to one another when the analysis is performed in terms of the sectors. Despite the fact that the highest and lowest numbers are not the same, the average values almost always stay the same. Banks, real estate investment, and service sectors are the industries with an average that is higher than other industries. One could state that this situation is very open to second-degree interactions in the stock pricing of relevant sectors on the basis of two-week trading periods during the stress period that began with the COVID-19 pandemic. Loans from financial institutions are a common source of funding for both real estate development initiatives and investment purchases of existing properties. To recoup their costs, banks collect interest payments from their customers and fees for services, such as financing, advice, and asset management. However, there are a number of ways in which the financial business can be impacted by the real estate financing field. When the real estate market declines, the institutions that retain real estate as security for lending can suffer substantial losses. Second, if real estate assets do not generate enough money, borrowers may not pay back their loans, which can hurt banks' finances. Finally, the service sector has a sizeable effect on financial institutions and real estate. If the service industry is thriving, for instance, there may be a rise in the need for bank loans to finance property growth and investment. Increased work and revenue from a flourishing service sector can also boost debtors' trustworthiness and lessen the likelihood of loan cancellations. Therefore, it is convenient to say that our findings are open to second-degree interactions.

Simple metals, chemicals, textiles, and transport are the industries that seem to have the widest average C_{P_3} distributions. The production of basic materials, intermediary products, and final products is the focus of the simple metals, chemicals, and textiles industries. The transport industry, on the other hand, is in charge with transporting people and products from one place to another. The following are ways in which these industries can be affected by their relationships with one another: First, basic materials, such as metals, chemicals, and textiles are essential to the production of products in the industries dealing with simple metals, chemicals, textile. Every industry's ground can be substantially affected by the cost and accessibility of these essential inputs. The quantity and price of basic materials are influenced by the transport industry, which is responsible for moving them from one location to another location. In addition, the price of labor, electricity, and transport can affect manufacturing expenses in industries such as simple metals, chemicals, or textile.

Manufacturing costs can be influenced by the price of personnel and electricity, while shipping costs can shape the final product pricing. The transport industry has a direct bearing on transport costs. Furthermore, the economy, customer tastes, and worldwide market patterns can all impact demand for goods in the industries dealing with basic metals, chemicals, textile. The revenue of the complete supply chain can be affected by the transport sector's performance in getting final goods to consumers. Moreover, the need for goods related to industries such as simple metals, chemicals, and textile can be significantly influenced by the creation of new technologies and advancements. Changes in operations and supply network can affect these businesses as much as new transit technologies. Hence, we conclude that pricing in these industries, which are susceptible to the monetary strain caused by the COVID-19 pandemic, particularly with regard to the logistics of raw material procurement, is accomplished in longer intervals between trade days.

To perform an analysis of the interaction between industries with regard to visibility graphs, individual visibility graphs that belonged to each sector were obtained by maintaining the same two-year period throughout the process, and by comparing graphs. A clustering analysis was carried out on the visibility graphs using a kernel function that was dependent on the C_{P_3} distribution. This was performed in addition to the first type of comparison, which was based on the topological aspects of graphs. When the distributions of the topological measures of the generated networks was analyzed, we found that, with the exception of information technologies, the C_{P_3} distributions of the other industries were relatively comparable to one another. In this scenario, it is plausible to assert that second-order connectivity plays a significant role in the daily pricing of the industries that comprise the BIST 100 index during the period of two years. In addition, the degree distributions may provide information that is analogous to what has been described. This problem also manifests itself in the kernel matrix, which provides a measurement of the degree to which the visibility graphs of different industries are the same. We performed hierarchical clustering over the kernel matrix, which was used to illustrate how comparable sectoral pricing was throughout the COVID-19 stress period. It has come to light that industries such as information technologies and service have a strong tendency toward clustering together. This is despite the fact that the service industry has been at the forefront of implementing productive technological developments in the Turkish economy in recent years. In addition, the industries regarding monetary resources are grouped into several clusters, ranked at the top. The only cluster that does not have an industry included in the monetary upper industries is cluster 3.

Food and beverage industry as well as transport are essential for Turkey's thriving tourism industry. Tourism is indisputably connected to the need for hotels, restaurants, and transport. Restaurants, coffee shops, clubs, and other food and beverage establishments may see an uptick in business as tourism expands, as may transport services such as car rentals, taxis, public transportation. The expansion of these industries can boost demand for labor and can supply tax money for public goods and services. In addition, Turkey's food and beverage industry is crucial and makes major contributions to the country's GDP. Numerous different establishments operate in this industry. Since visitors are always on the lookout for new and exciting dining opportunities, the tourism business is a driving force behind the expansion of the food and beverage industry. Therefore, an increase in the number of tourists may have a noticeable effect on the development of the food and beverage companies. The transport business is also crucial for the food and beverage industry since it is responsible for moving both materials and final goods. The expansion of the transit industry may have far-reaching consequences for hospitality and agriculture. Tourists will have an easier time getting around and finding places to eat if the public transport system is well-maintained and easy to use. Food and beverage delivery relies heavily on transport, which can affect products' accessibility and expenses. As a result of the organic connections existing between food and beverage industry and tourism industry, as well as textile and transport, one can state that the pricing characteristics registered in this cluster are comparable to one another.

Our results on the impact of the COVID-19 pandemic on Borsa Istanbul revealed important insights. According to the literature, one study [36] reported that the highest losses occurred in sports, tourism, and transport. In our case, based on the pricing analysis method that we advanced, the tourism and transport industries have a high-order relationship. Another study [49] examined the pricing analysis of Borsa Istanbul in the context of the 2008 global financial crisis and the Greek debt crisis. Prior to the 2008 global financial crisis, they identified reciprocal volatility spillovers between the industrial and financial sectors and a unidirectional volatility spillover from the services to the industrial sector. In addition, authors identified volatility spillovers from the industrial sector to the services and financial sectors, as well as from the services sector to the financial one. This situation was also consistent with the findings from our study.

When it comes to examining the links between various financial instruments and spotting trends and patterns in financial data, visibility graphs are a valuable tool. They have the potential to provide investors and academics who are researching financial markets insightful information. They can also be a significant instrument for risk management and decision-making in the financial sector. It is important that future research tries to investigate the efficiency of visibility graphs and their applicability in machine learning strategies for financial markets.

Author Contributions: Conceptualization, Ö.A., M.A.B. and L.M.B.; methodology, Ö.A. and M.A.B.; software, Ö.A. and M.A.B.; validation, Ö.A., M.A.B. and L.M.B.; formal analysis, Ö.A., M.A.B. and L.M.B.; investigation, Ö.A., M.A.B., L.M.B. and L.G.; resources, Ö.A., M.A.B., L.M.B. and L.G.; data curation, Ö.A. and M.A.B.; writing—original draft preparation, Ö.A., M.A.B., L.M.B. and L.G.; writing—review and editing, Ö.A., M.A.B., L.M.B. and L.G.; visualization, Ö.A. and M.A.B.; supervision, M.A.B. and L.M.B.; funding acquisition, L.G. All authors have read and agreed to the published version of the manuscript.

Funding: This study was conducted with financial support from the scientific research funds of the "1 Decembrie 1918" University of Alba Iulia, Romania.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data that have been used are presented in the manuscript with the relevant sources.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

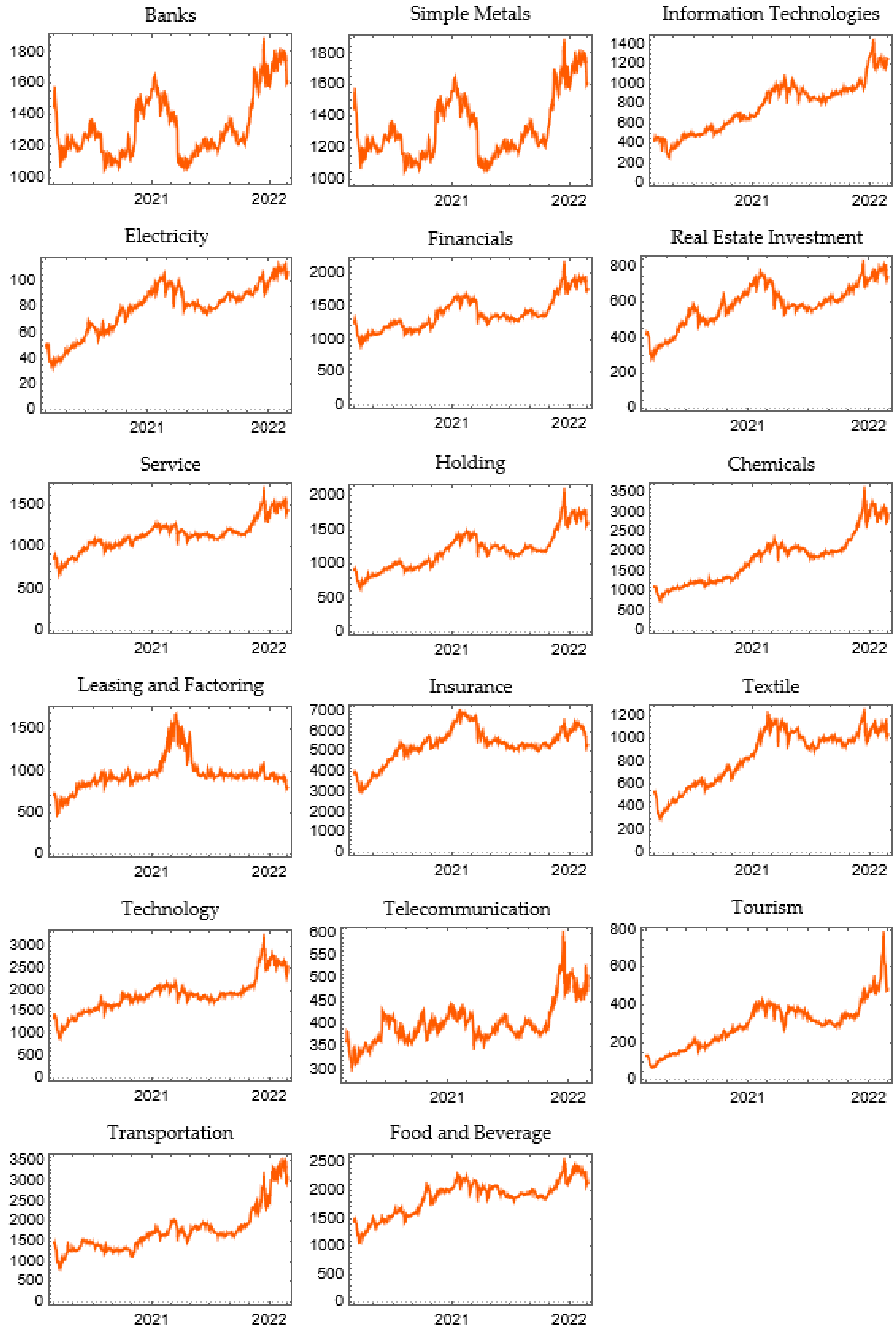

Figure A1. Time series data sets of each BIST 100 sector.

Table A1. Descriptive statistics for the data set.

Sector	Min	Mean	Max	Standard Deviation	Kurtosis	Skewness
Banks	1054.65	1335.9	1866.47	199.18	2.47647	0.757996
Simple Metals	1849.95	4424.34	9346.1	1780.42	2.50919	0.506485
Information Technologies	258.22	789.935	1413.88	262.54	2.11773	0.0626643
Electricity	33.96	78.9481	112.62	19.2967	2.44945	−0.561655
Financials	919.36	1398.12	2129.3	248.696	2.65921	0.602819
Real Estate Investment	288.83	590.945	820.23	119.53	2.74951	−0.475293
Service	678.69	1136.99	1663.21	180.363	3.30849	0.176691
Holding	654.82	1199.7	2049.26	274.344	2.77676	0.462593
Chemicals	772.89	1844.39	3555.92	627.604	2.47802	0.450528
Leasing and Factoring	478.33	961.712	1683.42	197.635	5.58505	1.22051
Insurance	3034.7	5346.99	6926.12	856.618	3.49947	−0.680855
Technology	300.42	850.387	1228.75	239.799	2.05888	−0.580769
Textile	923.81	1890.88	3177.55	398.207	3.58329	0.465362
Telecommunication	303.47	402.569	595.92	45.7838	4.48809	1.03093
Tourism	69.13	299.15	768.46	120.833	3.26537	0.303744
Transport	857.02	1776.68	3484.55	558.334	4.66538	1.41716
Food and Beverage	1061.92	1886.09	2516.27	307.04	2.63247	−0.521272

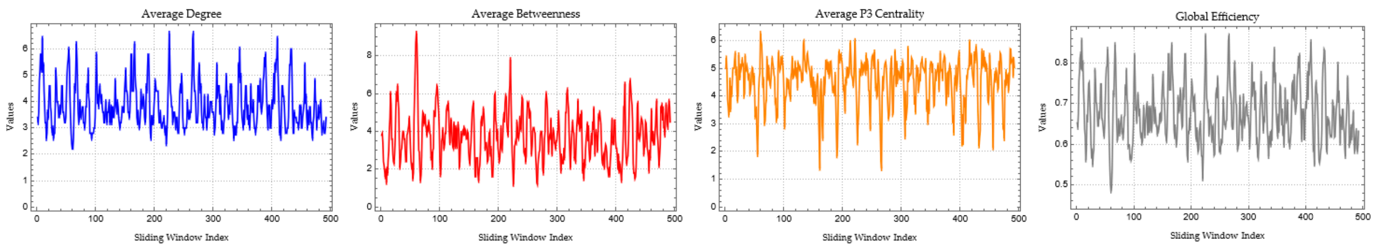

Figure A2. Results for Banks sector.

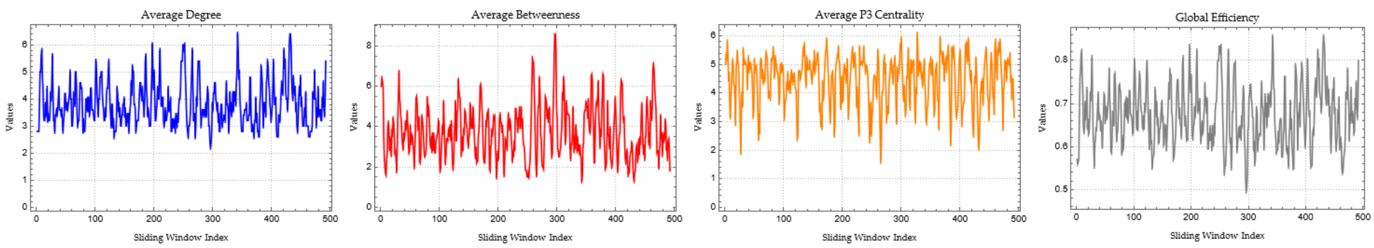

Figure A3. Results for Simple Metals sector.

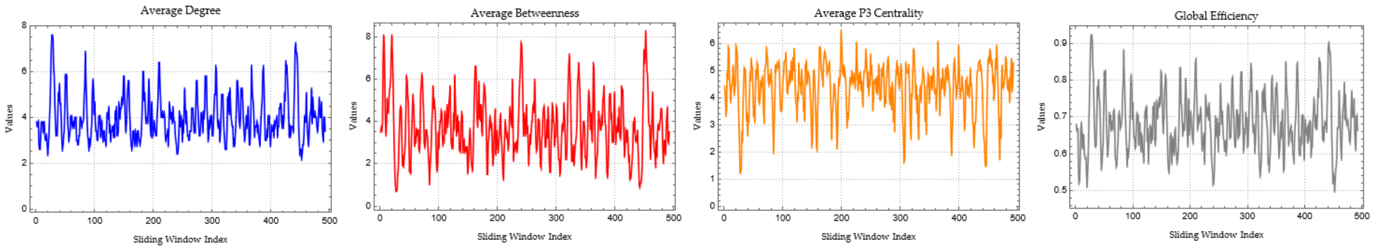

Figure A4. Results for Information Technologies sector.

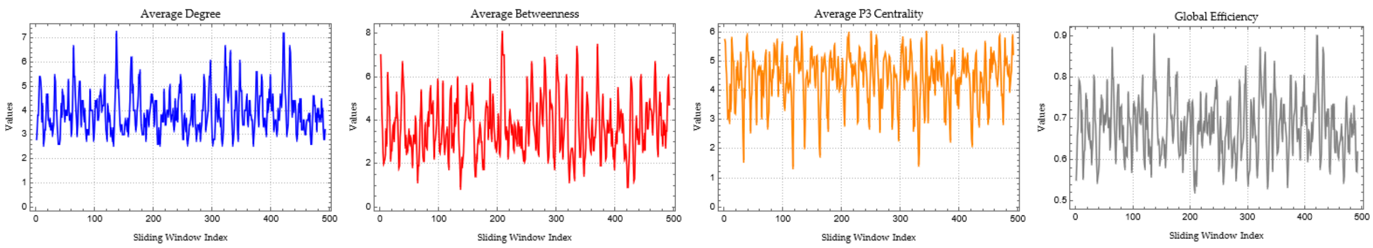

Figure A5. Results for Electricity sector.

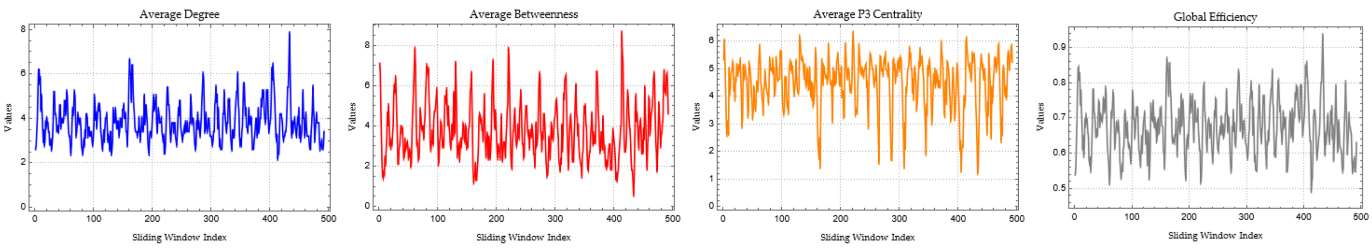

Figure A6. Results for Financials sector.

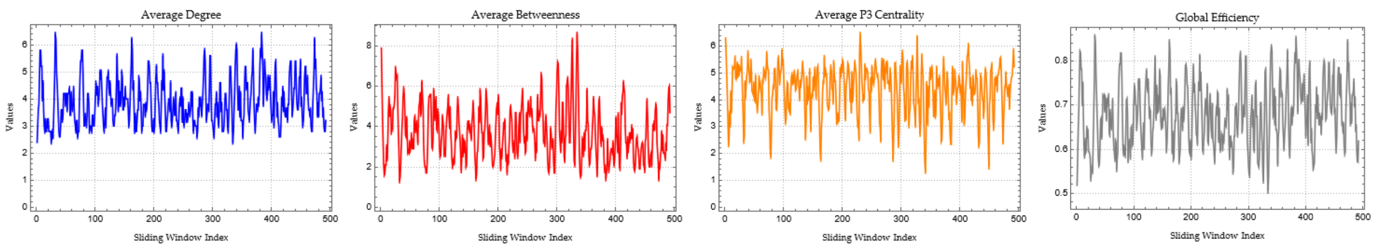

Figure A7. Results for Real Estate Investment sector.

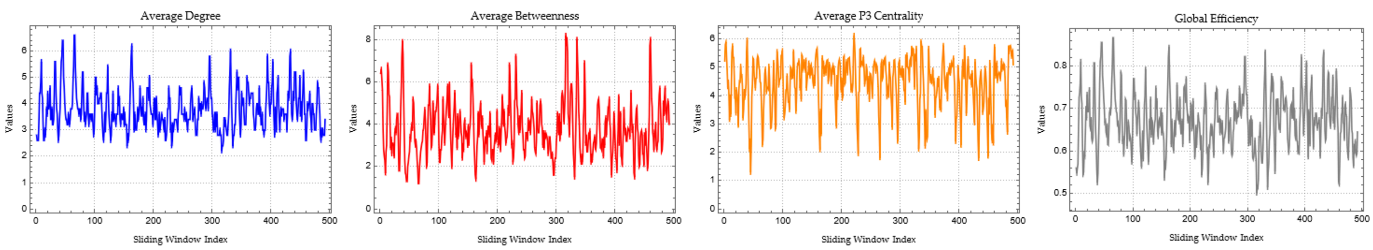

Figure A8. Results for Service sector.

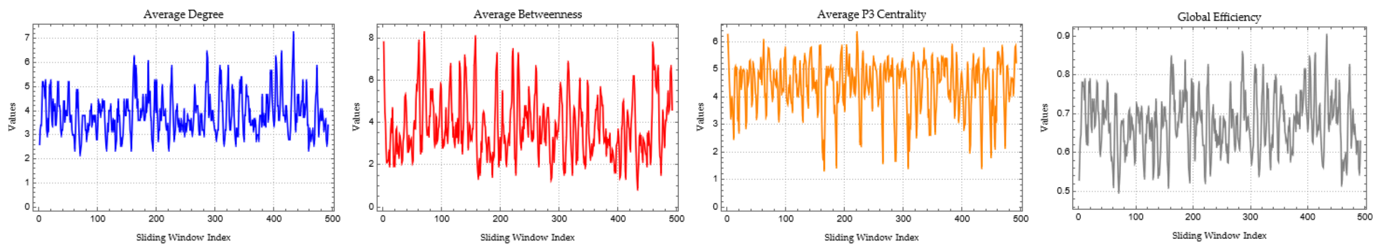

Figure A9. Results for Holding sector.

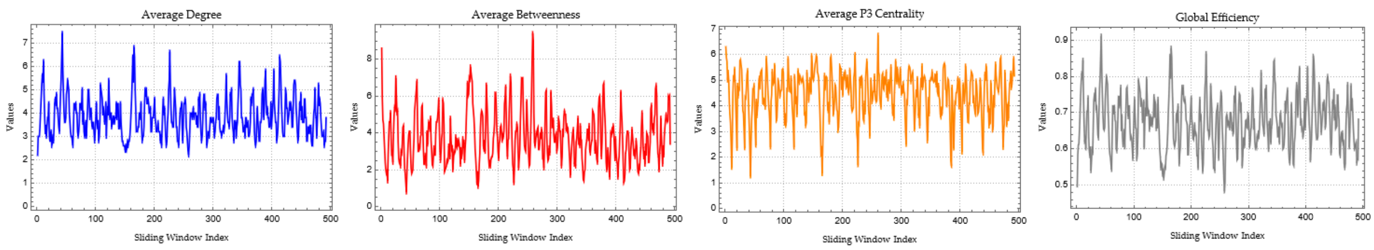

Figure A10. Results for Chemicals sector.

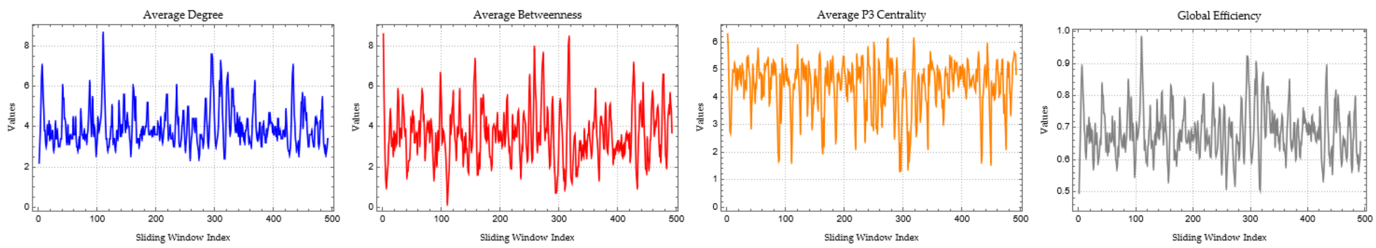

Figure A11. Results for Leasing and Factoring sector.

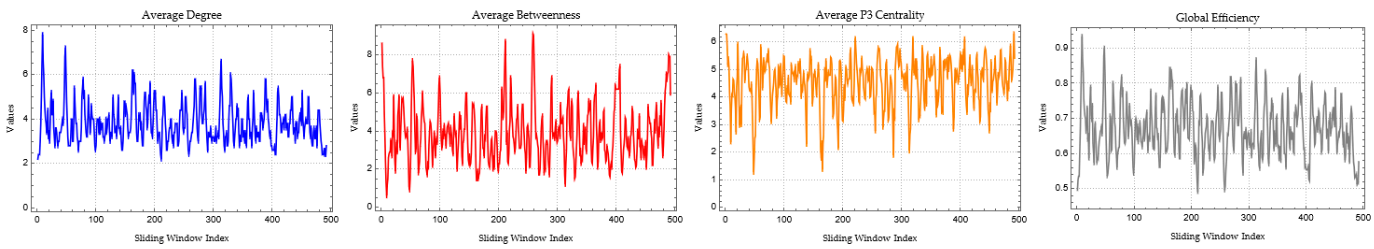

Figure A12. Results for Insurance sector.

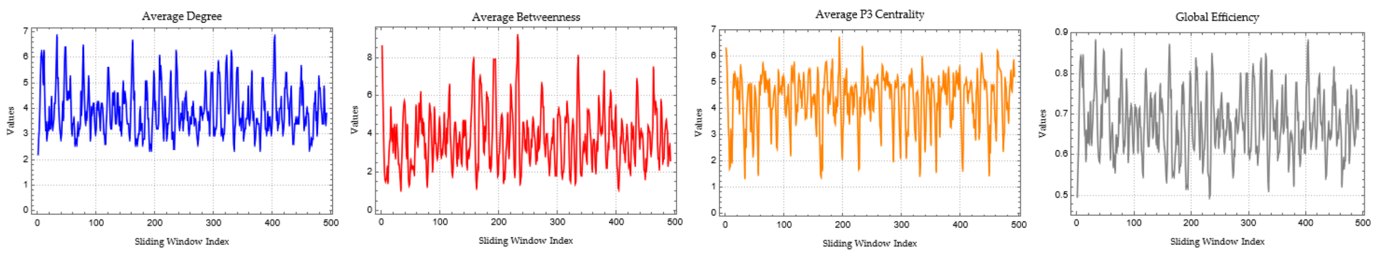

Figure A13. Results for Textile sector.

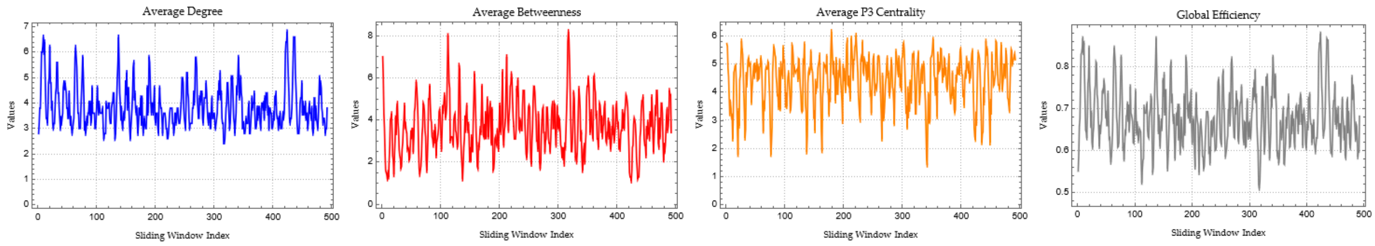

Figure A14. Results for Technology sector.

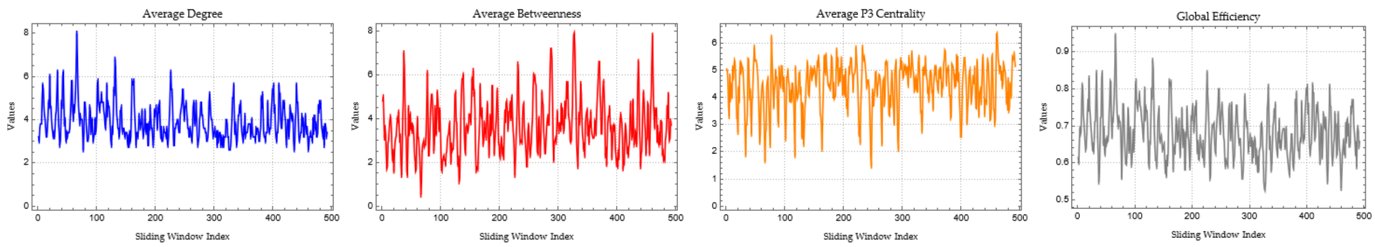

Figure A15. Results for Telecommunication sector.

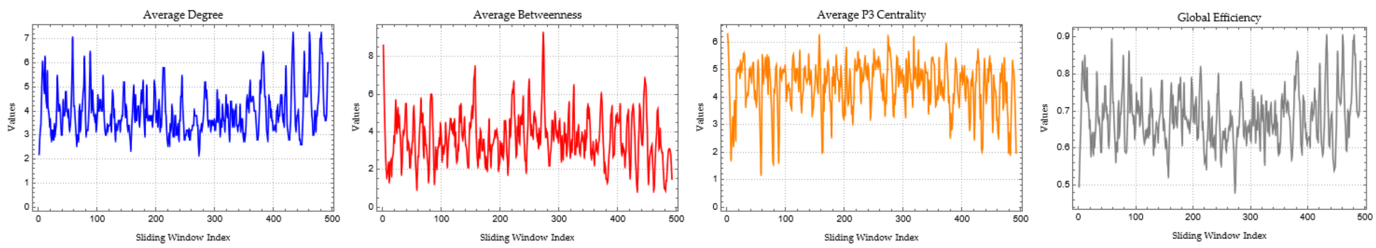

Figure A16. Results for Tourism sector.

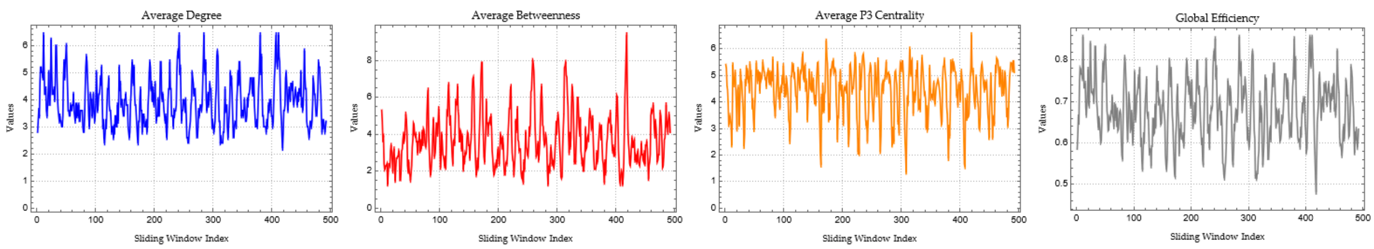

Figure A17. Results for Transport sector.

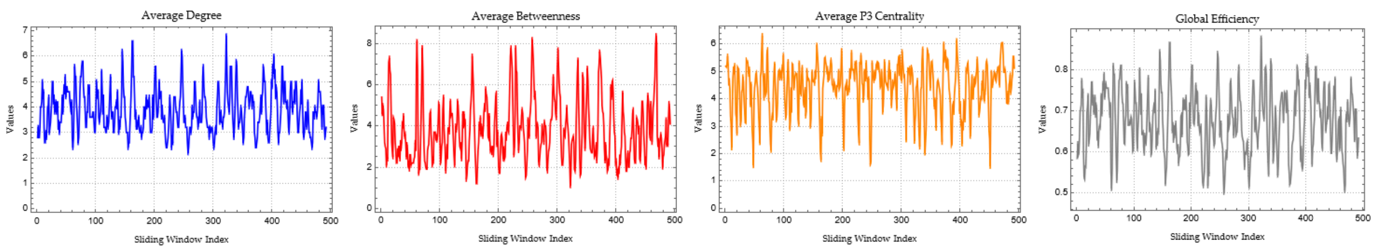

Figure A18. Results for Food and Beverage sector.

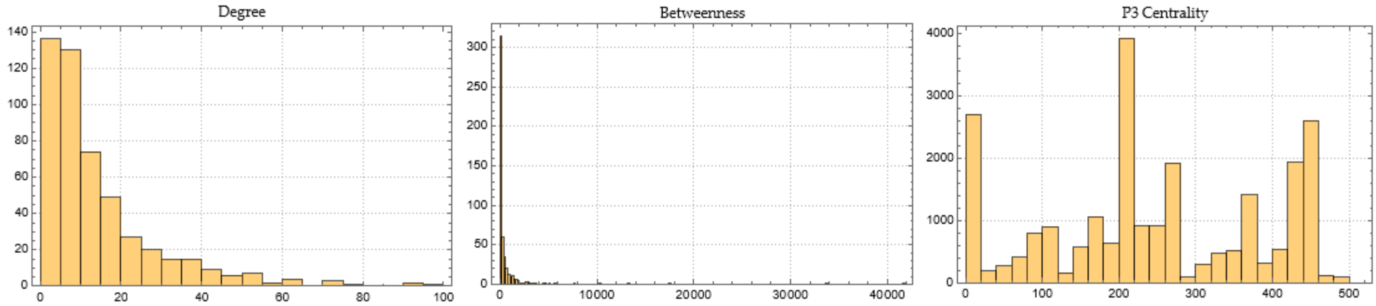

Figure A19. Distributions of topological measure of visibility graph emerging from Banks sector.

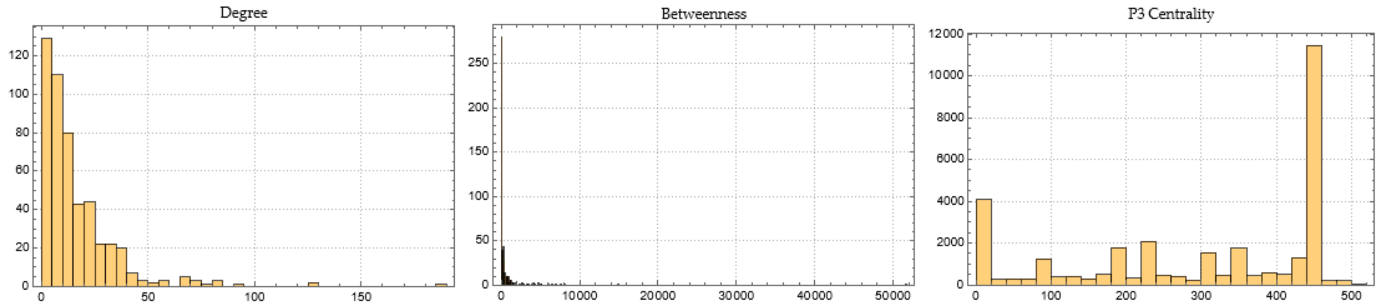

Figure A20. Distributions of topological measure of visibility graph emerging from Simple Metals sector.

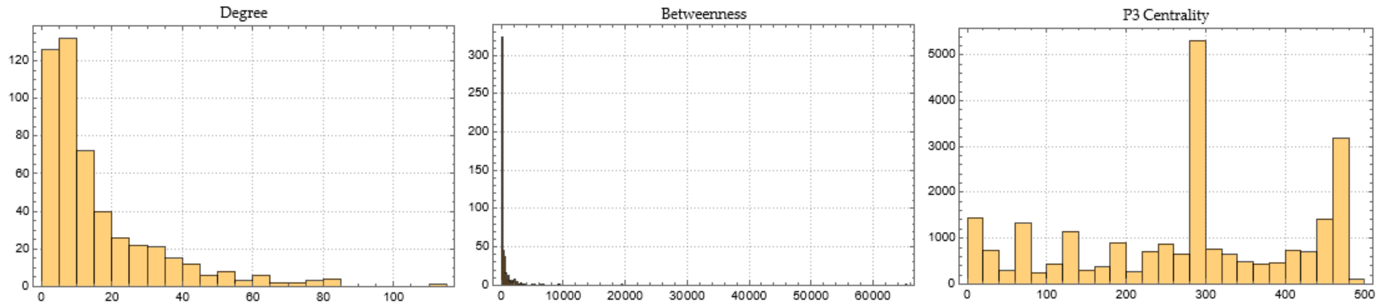

Figure A21. Distributions of topological measure of visibility graph emerging from Information Technologies sector.

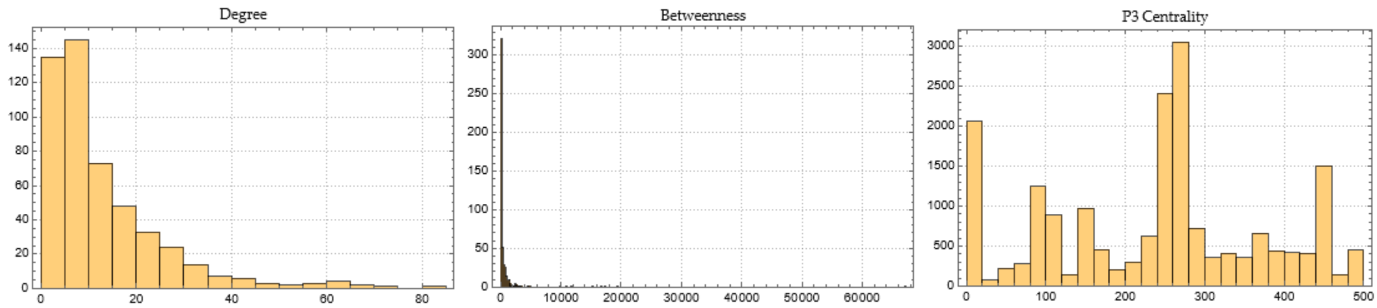

Figure A22. Distributions of topological measure of visibility graph emerging from Electricity sector.

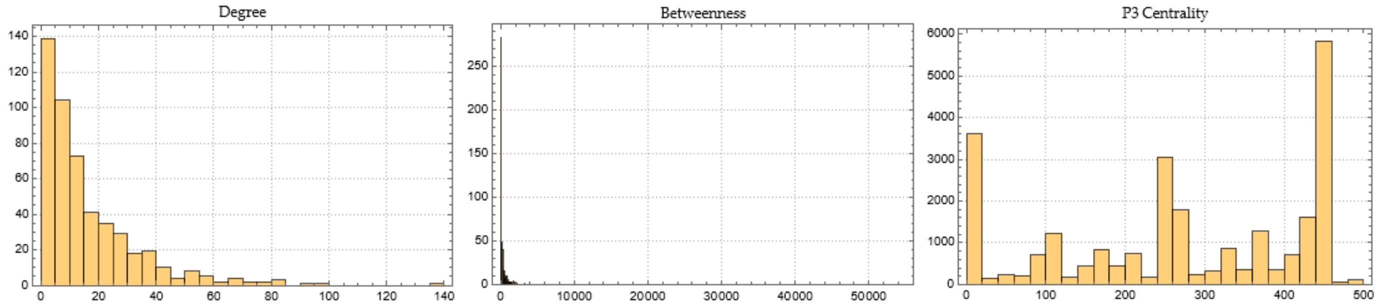

Figure A23. Distributions of topological measure of visibility graph emerging from Financials sector.

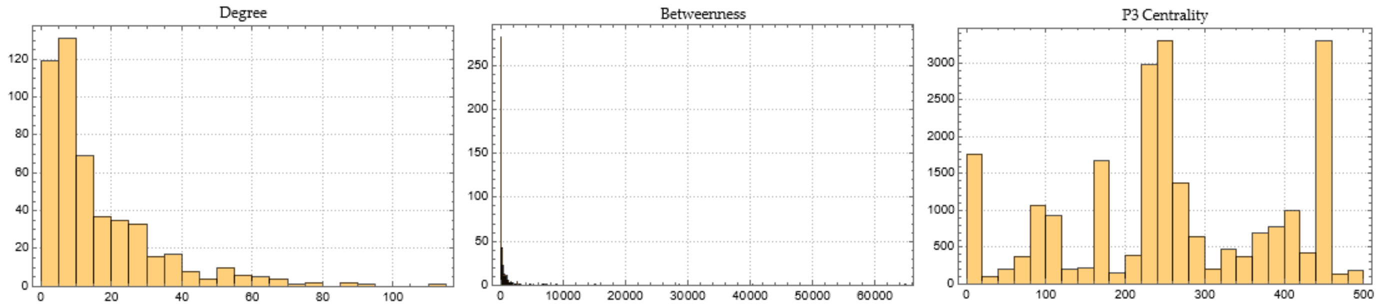

Figure A24. Distributions of topological measure of visibility graph emerging from Real Estate Investment sector.

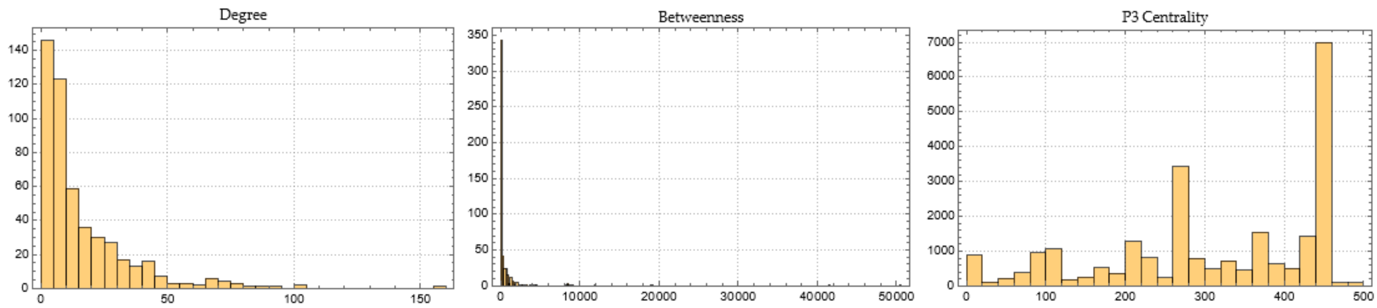

Figure A25. Distributions of topological measure of visibility graph emerging from Service sector.

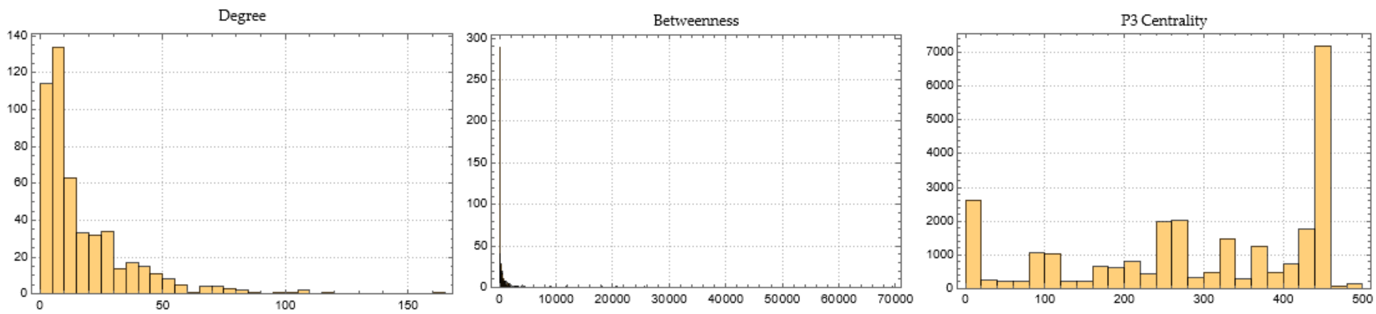

Figure A26. Distributions of topological measure of visibility graph emerging from Holding sector.

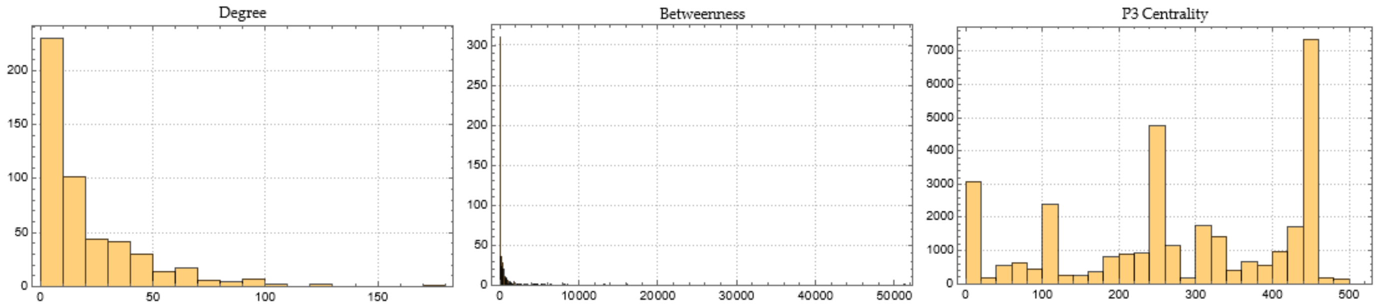

Figure A27. Distributions of topological measure of visibility graph emerging from Chemicals sector.

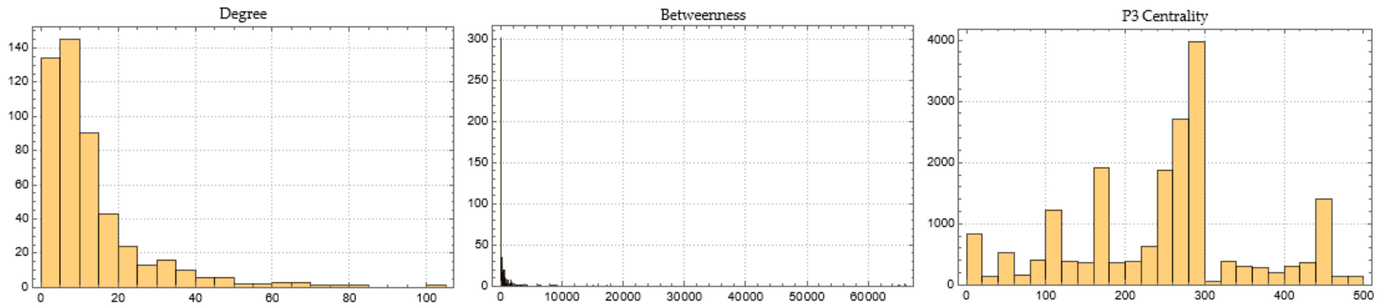

Figure A28. Distributions of topological measure of visibility graph emerging from Leasing and Factoring sector.

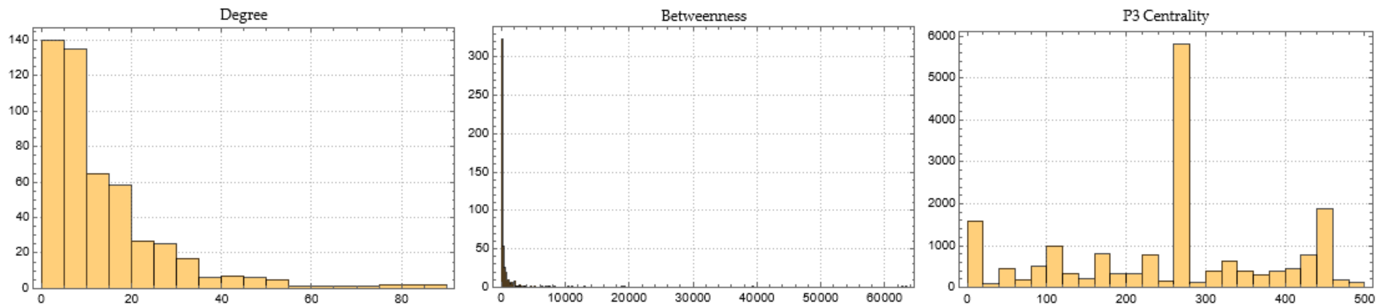

Figure A29. Distributions of topological measure of visibility graph emerging from Insurance sector.

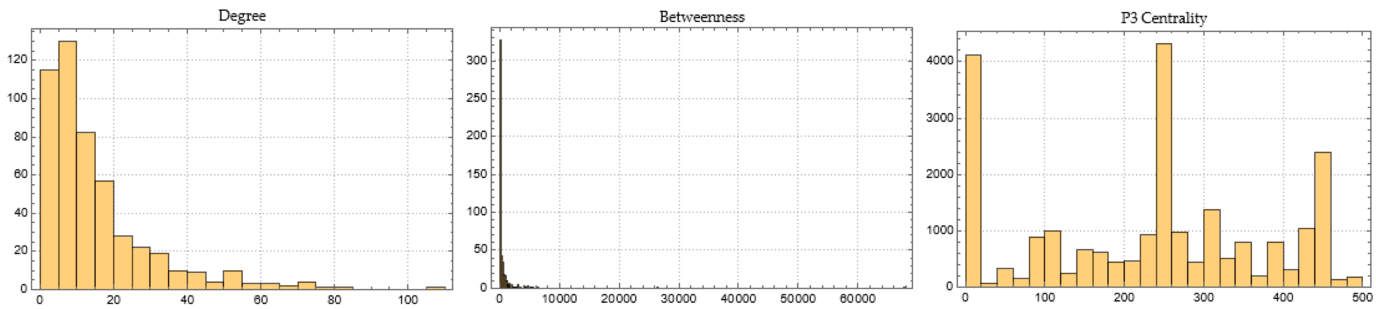

Figure A30. Distributions of topological measure of visibility graph emerging from Textile sector.

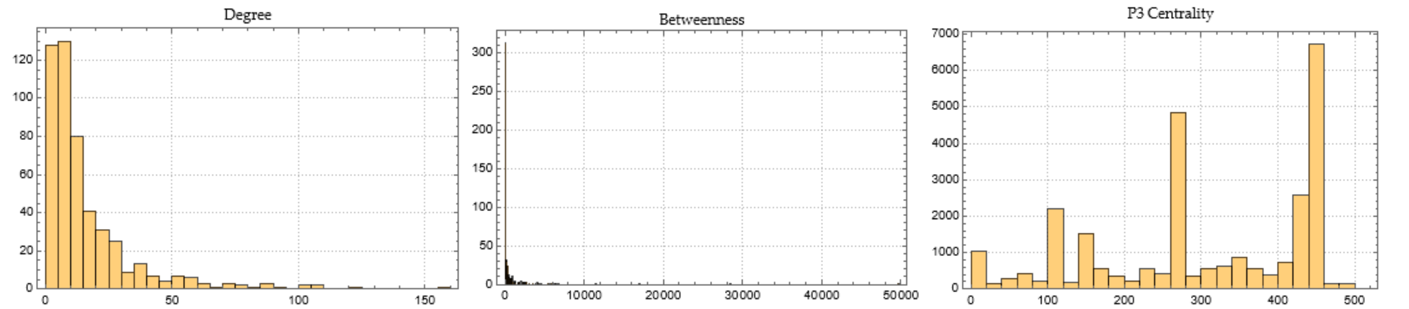

Figure A31. Distributions of topological measure of visibility graph emerging from Technology sector.

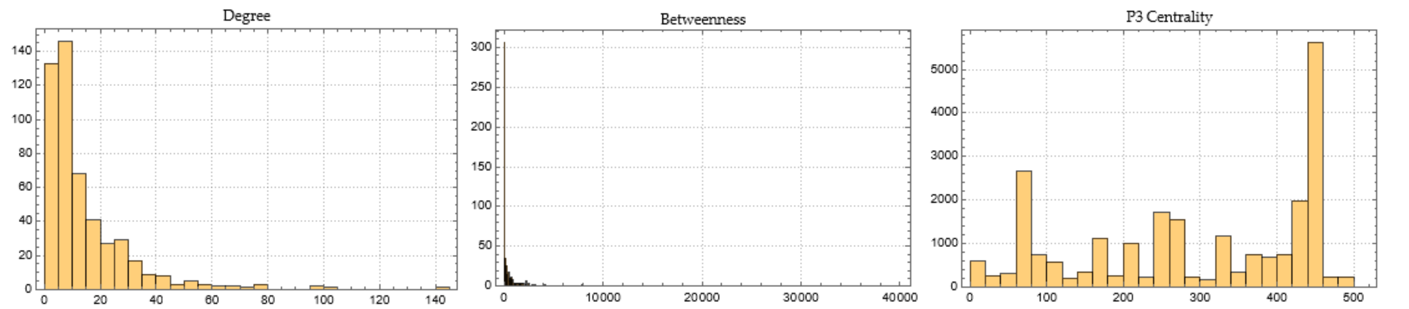

Figure A32. Distributions of topological measure of visibility graph emerging from Telecommunication sector.

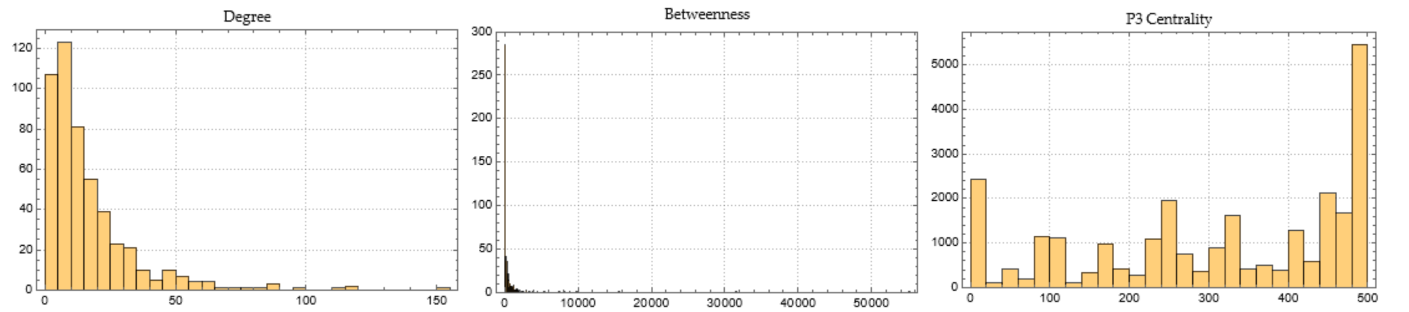

Figure A33. Distributions of topological measure of visibility graph emerging from Tourism sector.

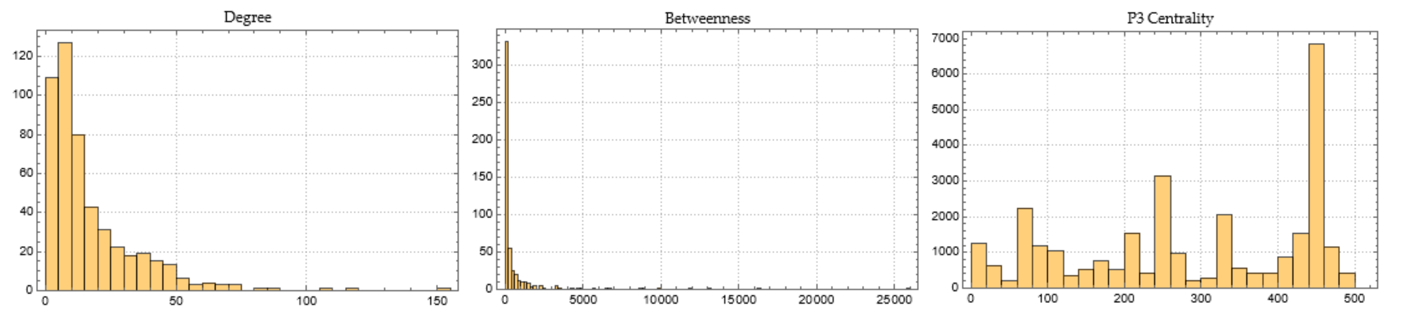

Figure A34. Distributions of topological measure of visibility graph emerging from Transport sector.

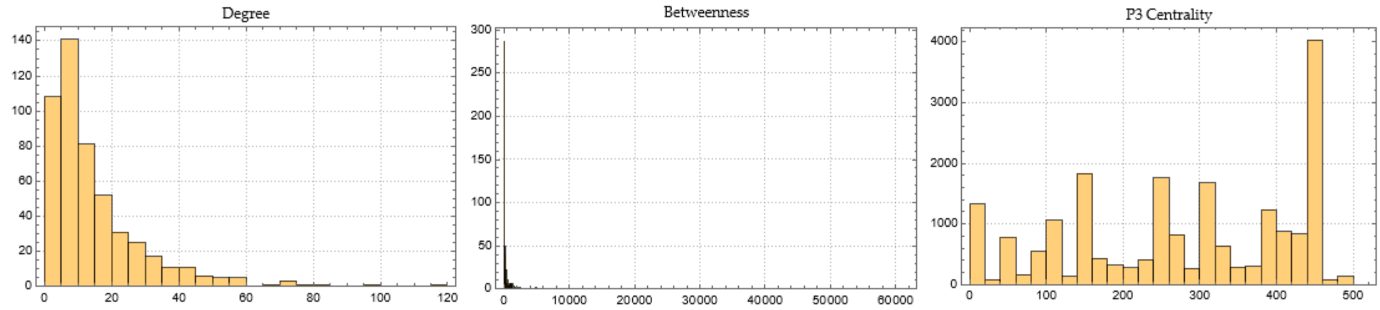

Figure A35. Distributions of topological measure of visibility graph emerging from Food and Beverage sector.

References

1. Lacasa, L.; Luque, B.; Ballesteros, F.; Luque, J.; Nuno, J.C. From time series to complex networks: The visibility graph. *Proc. Natl. Acad. Sci. USA* **2008**, *105*, 4972–4975. [CrossRef]
2. Lacasa, L.; Luque, B.; Luque, J.; Nuno, J.C. The visibility graph: A new method for estimating the Hurst exponent of fractional Brownian motion. *Europhys. Lett.* **2009**, *86*, 30001. [CrossRef]
3. Elsner, J.B.; Jagger, T.H.; Fogarty, E.A. Visibility network of United States hurricanes. *Geophys. Res. Lett.* **2009**, *36*, L16702. [CrossRef]
4. Yang, Y.; Wang, J.; Yang, H.; Mang, J. Visibility graph approach to exchange rate series. *Phys. A Stat. Mech. Its Appl.* **2009**, *388*, 4431–4437. [CrossRef]
5. Liu, C.; Zhou, W.X.; Yuan, W.K. Statistical properties of visibility graph of energy dissipation rates in three-dimensional fully developed turbulence. *Phys. A Stat. Mech. Its Appl.* **2010**, *389*, 2675–2681. [CrossRef]
6. Shao, Z.G. Network analysis of human heartbeat dynamics. *Appl. Phys. Lett.* **2010**, *96*, 073703. [CrossRef]
7. Dong, Z.; Li, X. Comment on network analysis of human heartbeat dynamics. *Appl. Phys. Lett.* **2010**, *96*, 073703. [CrossRef]
8. Ahmadlou, M.; Adeli, H.; Adeli, A. New diagnostic EEG markers of the Alzheimer's disease using visibility graph. *J. Neural Transm.* **2010**, *117*, 1099–1109. [CrossRef]
9. Qian, M.C.; Jiang, Z.Q.; Zhou, W.X. Universal and nonuniversal allometric scaling behaviors in the visibility graphs of world stock market indices. *J. Phys. A Math. Theor.* **2010**, *43*, 335002. [CrossRef]
10. Ni, X.H.; Jiang, Z.Q.; Zhou, W.X. Degree distributions of the visibility graphs mapped from fractional Brownian motions and multifractal random walks. *Phys. Lett. A* **2009**, *373*, 3822–3826. [CrossRef]
11. Zhang, J.; Small, M. Complex network from pseudoperiodic time series: Topology versus dynamics. *Phys. Rev. Lett.* **2006**, *96*, 238701. [CrossRef] [PubMed]
12. Yang, Y.; Yang, H. Complex network-based time series analysis. *Phys. A Stat. Mech. Its Appl.* **2008**, *387*, 1381–1386. [CrossRef]
13. Xu, X.; Zhang, J.; Small, M. Superfamily phenomena and motifs of networks induced from time series. *Proc. Natl. Acad. Sci. USA* **2008**, *105*, 19601–19605. [CrossRef]
14. Gao, Z.; Jin, N. Erratum: "Complex network from time series based on phase space reconstruction". *Chaos Interdiscip. J. Nonlinear Sci.* **2010**, *20*, 019902. [CrossRef]
15. Donner, R.V.; Small, M.; Donges, J.F.; Marwan, N.; Zou, Y.; Xiang, R.; Kurths, J. Recurrence-based time series analysis by means of complex network methods. *Int. J. Bifurc. Chaos* **2011**, *21*, 1019–1046. [CrossRef]
16. Shirazi, A.H.; Jafari, G.R.; Davoudi, J.; Peinke, J.; Tabar, M.R.R.; Sahimi, M. Mapping stochastic processes onto complex networks. *J. Stat. Mech. Theory Exp.* **2009**, *2009*, P07046. [CrossRef]
17. Berg, M.D.; Kreveld, M.V.; Overmars, M.; Schwarzkopf, O.C. Visibility Graphs. In *Computational Geometry*; Springer: Berlin/Heidelberg, Germany, 2000; pp. 307–317.
18. Battiston, S.; Caldarelli, G.; May, R.M.; Roukny, T.; Stiglitz, J.E. The price of complexity in financial networks. *Proc. Natl. Acad. Sci. USA* **2016**, *113*, 10031–10036. [CrossRef]
19. Junior, L.; Mullokandov, A.; Kenett, D. Dependency relations among international stock market indices. *J. Risk Financ. Manag.* **2015**, *8*, 227–265. [CrossRef]
20. Barucca, P.; Bardoscia, M.; Caccioli, F.; D'Errico, M.; Visentin, G.; Caldarelli, G.; Battiston, S. Network valuation in financial systems. *Math. Financ.* **2020**, *30*, 1181–1204. [CrossRef]
21. Gao, Q. Systemic risk analysis of multi-layer financial network system based on multiple interconnections between banks, firms, and assets. *Entropy* **2022**, *24*, 1252. [CrossRef]
22. Su, Y.; Huang, Z.; Drakeford, B.M. Monetary policy, industry heterogeneity and systemic risk—Based on a high dimensional network analysis. *Sustainability* **2019**, *11*, 6222. [CrossRef]
23. Dastkhan, H.; Gharneh, N.S. Determination of systemically important companies with cross-shareholding network analysis: A case study from an emerging market. *Int. J. Financ. Stud.* **2016**, *4*, 13. [CrossRef]

24. Seong, N.; Nam, K. Forecasting price movements of global financial indexes using complex quantitative financial networks. *Knowl.-Based Syst.* **2022**, *235*, 107608. [CrossRef]
25. Kou, G.; Chao, X.; Peng, Y.; Wang, F. Network resilience in the financial sectors: Advances, key elements, applications, and challenges for financial stability regulation. *Technol. Econ. Dev. Econ.* **2022**, *28*, 531–558. [CrossRef]
26. Balcı, M.A.; Batrancea, L.M.; Akgüller, Ö. Network-induced soft sets and stock market applications. *Mathematics* **2022**, *10*, 3964. [CrossRef]
27. Aslam, F.; Mohmand, Y.T.; Ferreira, P.; Memon, B.A.; Khan, M.; Khan, M. Network analysis of global stock markets at the beginning of the coronavirus disease (COVID-19) outbreak. *Borsa Istanb. Rev.* **2020**, *20*, S49–S61. [CrossRef]
28. Balcı, M.A. Hierarchies in communities of Borsa Istanbul stock exchange. *Hacet. J. Math. Stat.* **2018**, *47*, 921–936.
29. Baydilli, Y.Y.; Bayir, Ş.; Türker, I. A hierarchical view of a national stock market as a complex network. *Econ. Comput. Econ. Cybern. Stud. Res.* **2017**, *51*, 205–222.
30. Akgüller, Ö.; Balcı, M.A. Geodetic convex boundary curvatures of the communities in stock market networks. *Phys. A Stat. Mech. Its Appl.* **2018**, *505*, 569–581. [CrossRef]
31. Akgüller, Ö. A threshold method for financial networks and geometric scattering of agents. *Commun. Stat. Case Stud. Data Anal. Appl.* **2019**, *5*, 230–242. [CrossRef]
32. Şükrüoğlu, D. Effects of COVID-19 on the BIST 100 network structure. *Appl. Econ.* **2022**, *54*, 5991–6007. [CrossRef]
33. Demiral, D.G.; Değirmenci, N. Evaluation of Borsa İstanbul with social network analysis method. *Int. J. Contemp. Econ. Adm. Sci.* **2021**, *11*, 060–075.
34. Özkan, O. Volatility jump: The effect of COVID-19 on Turkey stock market. *Gaziantep Univ. J. Soc. Sci.* **2020**, *19*, 386–397. [CrossRef]
35. Balcı, M.A.; Batrancea, L.M.; Akgüller, Ö.; Gaban, L.; Rus, M.I.; Tulai, H. Fractality of Borsa Istanbul during the COVID-19 Pandemic. *Mathematics* **2022**, *10*, 2503. [CrossRef]
36. Göker, İ.E.K.; Eren, B.S.; Karaca, S.S. The impact of the COVID-19 (Coronavirus) on the Borsa Istanbul sector index returns: An event study. *Gaziantep Univ. J. Soc. Sci.* **2020**, *19*, 14–41.
37. Tan, Ö.F. The impact of news about pandemic on Borsa Istanbul during the COVID-19 financial turmoil. *Türkiye İletişim Araştırmaları Derg.* **2021**, *37*, 109–124.
38. Atici, G.; Gursoy, G. Trends of non-financial corporations listed on Borsa Istanbul: Rethinking corporate ownership and governance under COVID-19. *J. Gov. Regul.* **2020**, *9*, 132–143. [CrossRef]
39. Yağlı, İ. The impact of COVID-19 on emerging stock market volatility: Empirical evidence from Borsa Istanbul. *Ekon. Polit. Finans. Araştırmaları Derg.* **2020**, *5*, 269–279.
40. Erdogan, H.H. Beta herding in the COVID-19 era: Evidence from Borsa Istanbul. *Bus. Econ. Res. J.* **2021**, *12*, 359–368. [CrossRef]
41. Lan, X.; Mo, H.; Chen, S.; Liu, Q.; Deng, Y. Fast transformation from time series to visibility graphs. *Chaos Interdiscip. J. Nonlinear Sci.* **2015**, *25*, 083105. [CrossRef]
42. Wu, J.; Xu, K.; Chen, X.; Li, S.; Zhao, J. Price graphs: Utilizing the structural information of financial time series for stock prediction. *Inf. Sci.* **2022**, *588*, 405–424. [CrossRef]
43. Moreira, F.R.D.S.; Verri, F.A.N.; Yoneyama, T. Maximum visibility: A novel approach for time series forecasting based on complex network theory. *IEEE Access* **2022**, *10*, 8960–8973. [CrossRef]
44. Tandon, H.; Ranjan, P.; Chakraborty, T.; Suhag, V. Coronavirus (COVID-19): ARIMA-based time-series analysis to forecast near future and the effect of school reopening in India. *J. Health Manag.* **2022**, *24*, 373–388.
45. Mehtab, S.; Sen, J. Analysis and Forecasting of Financial Time Series Using CNN and LSTM-Based Deep Learning Models. In *Advances in Distributed Computing and Machine Learning*; Springer: Singapore, 2022; pp. 405–423.
46. Kaibuchi, H.; Kawasaki, Y.; Stupfler, G. GARCH-UGH: A bias-reduced approach for dynamic extreme value-at-risk estimation in financial time series. *Quant. Financ.* **2022**, *22*, 1277–1294. [CrossRef]
47. Cui, X.; Hu, J.; Ma, Y.; Wu, P.; Zhu, P.; Li, H.J. Investigation of stock price network based on time series analysis and complex network. *Int. J. Mod. Phys. B* **2021**, *35*, 2150171. [CrossRef]
48. Donner, R.V.; Donges, J.F.; Zou, Y.; Feldhoff, J.H. Complex Network Analysis of Recurrences. In *Recurrence Quantification Analysis*; Springer: Cham, Switzerland, 2015; pp. 101–163.
49. Kamışlı, M.; Kamışlı, S.; Sevil, G. The effects of crises on volatility spillovers between Borsa Istanbul sector indexes. *Adv. Econ. Bus.* **2016**, *4*, 339–344. [CrossRef]