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Buckling assessment of cylindrical steel tanks with top stiffening ring under wind loading

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ABSTRACT: A stiffening ring is commonly used at the top of the tank wall to increase its strength against external pressure instability. Traditional design treatments generally consider cylindrical storage tanks under uniform external pressure for sizing of the top ring. However, cylindrical steel tanks under non-uniform wind loading have rather different and complex buckling behaviour from those of tanks subjected to uniform external wind loading. In this study, the buckling resistance of the cylindrical steel storage tanks with top stiffening ring under wind loading is investigated using finite element analyses. The changes in the buckling capacity are studied in light of the proposed stiffness ratio for a particular harmonic of wind loading. The results revealed that the changes in the buckling capacity are closely related to the shell-top ring stiffness ratio. Furthermore, a generalized solution that shows buckling pressure ratio ($q_{cr,w}/q_{cr,D}$) is then developed.

1 INTRODUCTION

Cylindrical ground-supported storage tanks are widely used to store a great variety of liquids for both short and long term purposes. A very common failure mode of such a storage tank is under wind loading, where the tank wall may have the insufficient strength and stiffness to resist external pressure. The wall thickness of a storage tank is normally chosen to resist only the internal pressure from the stored product. Storage tanks are susceptible to buckling under wind load when they are empty or at a low-level of filling (Ansourian 1992, Flores & Godoy 1998, Maraveas et al. 2015). One way to strengthen the tank wall is to use a stiffening ring placed at the top or near the top of the tank (Figure 1). This ring also plays an important role in maintaining circularity when the tank is subjected to wind loads. The circularity is particularly important when a floating roof is used within the tank, and this condition also means that there is no structural roof to resist buckling displacements at the top of the wall. Because the tank is very thin, it is very susceptible to buckling under external pressure, and under wind this is exacerbated because the pressure varies significantly around the circumference, flattening the wall locally and inducing stresses in different directions (Maher 1966). Thus, the specific pattern of pressure variation around circumference is of great importance in design against wind. Traditional design treatments generally consider cylindrical storage tanks under uniform external pressure for the sizing of the top ring. However, cylindrical steel tanks under non-uniform wind loading have rather different and complex buckling behaviour from those of tanks subjected to uniform external wind loading (Chen & Rotter 2012).

The stiffening ring should be designed against buckling under external pressure. For this purpose, this ring must have an adequate stiffness to fulfil its function. Buckling of ring stiffened cylinders under non-uniform wind loading is a rather difficult problem in shell analysis.

Blackler (1986) proposed an expression for the minimum stiffness requirement of top stiffening ring under uniform external pressure as follows:

$$I_{r,\min} = 0.048Lt^3 \tag{1}$$

where L = length of the cylindrical shell; t = thickness of the shell wall, $I_{r,\min}$ = minimum moment of inertia of the ring.

Schmidt (1998) also suggested a minimum moment of inertia for the top ring based on the post-buckling behaviour under external pressure.

$$I_{r,\min} = 0.5Lt^3 \tag{2}$$

Greiner & Guggenberger (2004) identified a different requirement for the limiting stiffness, which this value was about 10 times the proposal of Blackler (1986).

$$I_{r,\min} = 0.48Lt^3 \tag{3}$$

Eurocode EN 1993-4-1 (2007) requires both a strength and a stiffness requirement for the top ring. It has a moment capacity requirement to address the effect of uniform external pressure on a ring that is imperfectly round, as well as a further requirement for wind conditions. A further requirement is for the flexural rigidity of the ring, intended to ensure that the ring does not participate in the buckling mode if the shell wall buckles. The expressions for the flexural rigidity of the ring about its vertical axis were given as follows:

$$EI_{r,\min} = k_1 ELt^3 \tag{4}$$

$$EI_{r,\min} = 0.08C_w Ert^3\sqrt{r/t} \tag{5}$$

where C_w = the wind pressure distribution coefficient, r = radius of the shell, E = modulus of elasticity, k_1 = 0.1 is recommended.

According to this specification, the bending stiffness of the girder should be sufficiently large to restrict the out-of-round displacement under wind action of the ring stiffened cross-section to 2% of the radius (ECCS 2014). The above recommendations present a wide range of required minimum stiffness values for the ring size.

There are two potential requirements for the top stiffening ring: strength to ensure that the ring does not yield under the unsymmetrical loads that will be applied to it under wind, and the stiffness to ensure that the buckling assessment of the tank wall under wind can be based on complete radial restraint at the top. This paper addresses the latter using a more thorough analysis than any found in previous studies. The buckling resistance of the cylindrical steel storage tanks with top stiffening ring under wind loading that varies along the circumference is investigated using finite element analyses. A shell-ring stiffness ratio is devised algebraically from the relative radial stiffnesses of the ring and the cylindrical shell under each harmonic of wind loading. The changes in the buckling capacity are studied in light of the proposed stiffness ratio.

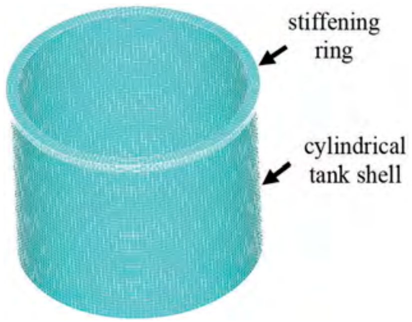

Figure 1. Rendering of tank and stiffening ring system.

2 WIND PRESSURE ON CIRCULAR CYLINDERS

The wind pressure distribution around a cylindrical shell has been studied extensively using wind tunnel tests (Maher 1966, Purdy et al. 1967, Resinger & Greiner 1982, MacDonald et al. 1988, Uematsu et al. 2018) where significant amounts of experimental data have been collected, leading to the characteristic pattern. The wind distribution for an isolated cylindrical structure depends on many parameters like the Reynolds number of the wind flow, the cylinder aspect ratio and potentially the shape of the roof (Maher 1966). The wind pressure varies both up the height and around the circumference of a cylindrical shell structure. However, the vertical pressure variation on the cylinder is usually assumed to be constant for tank structures because the aspect ratio is relatively low, leading to a relatively small vertical variation in the pressure (Bu & Qian 2016, Shokrzadeh & Sohrabi 2016) when compared with the major circumferential variation. Circumferential distribution of wind pressure on a circular tank structure can be reasonably approximated by a Fourier harmonic cosine series of the form as follows (EN 1993-4-1 2007):

$$\begin{aligned} C_p(\theta) = \sum_{m=0}^{4} a_i \cos(m\theta) &= -0.54 + 0.16(d_c/L) + \{0.28 + 0.04(d_c/L)\}\cos\theta \\ &+ \{1.04 - 0.20(d_c/L)\}\cos 2\theta + \{0.36 - 0.05(d_c/L)\}\cos 3\theta \\ &- \{0.14 - 0.05(d_c/L)\}\cos 4\theta \end{aligned} \tag{6}$$

where θ = the circumferential angle measured from the stagnation meridian, d_c = the diameter of the cylindrical shell, a_i = coefficients of each harmonic, m = the harmonic number.

The range of wind pressure distributions considering different tank aspect ratios given in the Eurocode standard EN 1993-4-1 (2007) is shown in Figure 2. The expression given in Equation 6 considers significant effects of the aspect ratio of the cylindrical structures.

Small changes in the aspect ratio may considerably alter the wind pressure profile. But, this pressure distribution changes little as the length increased for intermediate and tall cylindrical structures.

3 SHELL-RING STIFFNESS RATIO

The wind loading is principally resisted by membrane shear in the shell which transmits the translational force to the support. But because it contains loading components in higher harmonics than m=1, it also induces bending in the top ring and in the shell according to their relative stiffnesses. The ring must have an adequate stiffness to fulfil its function. A test for its adequacy was developed by comparing the relative stiffnesses of the ring and the shell under non-uniform load.

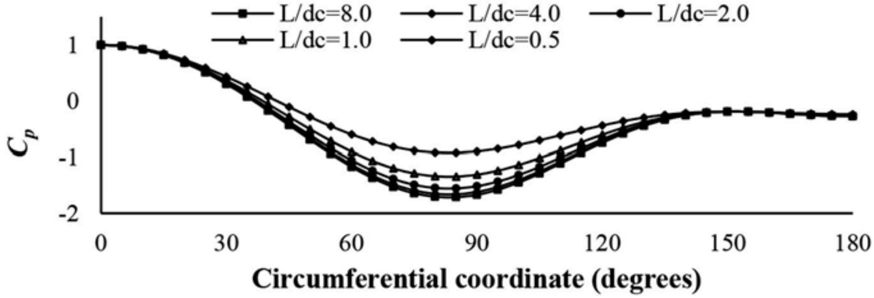

Figure 2. Wind pressure distribution around circumference for circular structures with different aspect ratios (EN 1993-4-1, 2007).

$$\chi = \frac{K_{shell}}{K_{ring}} = \frac{q_r(\theta)/u_{r,shell}}{q_r(\theta)/u_{r,ring}} = \frac{u_{r,ring}}{u_{r,shell}} \tag{7}$$

where K_{shell} and K_{ring} are radial stiffnesses of the shell and the top ring respectively; $u_{r,ring}$ and $u_{r,shell}$ are radial displacements of the ring and shell respectively, $q_r(\theta)$= the circumferential variation of the non-uniform line load considering wind pressure ($\bar{q}$) as defined by Equation 8.

$$q_r(\theta) = \bar{q} \times L \times a_i \cos m\theta \tag{8}$$

For any single harmonic loading, closed-form expressions were obtained for the radial displacement of the shell and stiffening ring.

3.1 *Ring beam stiffness*

The Vlasov curved beam differential equations (Vlasov 1961, Heins 1975) were used to study the response of the top ring. The equilibrium equations were first derived for the curved beam element shown in Figure 3, where three orthogonal internal forces and three internal moments develop at each cross-section. The six basic equilibrium equations can be expressed as follows:

$$\frac{1}{r}\left[\frac{dQ_r}{d\theta} + Q_\theta\right] + q_r = 0 \quad \frac{1}{r}\frac{dQ_x}{d\theta} + q_x = 0 \tag{9}$$

$$\frac{1}{r}\left[\frac{dQ_\theta}{d\theta} - Q_r\right] + q_\theta = 0 \quad \frac{1}{r}\left[\frac{dM_r}{d\theta} + T_\theta\right] - Q_x + m_r = 0 \tag{10}$$

$$\frac{1}{r}\frac{dM_x}{d\theta} + m_x + Q_r = 0 \quad \frac{1}{r}\left[\frac{dT_\theta}{d\theta} - M_r\right] + m_\theta = 0 \tag{11}$$

where M_r = bending moment in the ring about a radial axis; M_x = bending moment in the ring about a transverse axis; T_θ = torsional moment in the ring; q_x, q_θ, q_r = distributed line loads per unit length in the transverse; circumferential and radial directions respectively; m_x, m_θ, m_r = distributed applied torques per unit circumference about the transverse, circumferential and radial directions respectively; Q_θ = circumferential force in the ring; Q_x, Q_r = shear forces in the ring in transverse and radial directions respectively.

The differential equation for bending of the ring in its own plane can be uncoupled from the other two. For the case of wind loading, only radial loads q_r are needed (i.e. $q_\theta = q_x = m_r = m_\theta = m_x = 0$), and the uncoupled differential equation of equilibrium becomes:

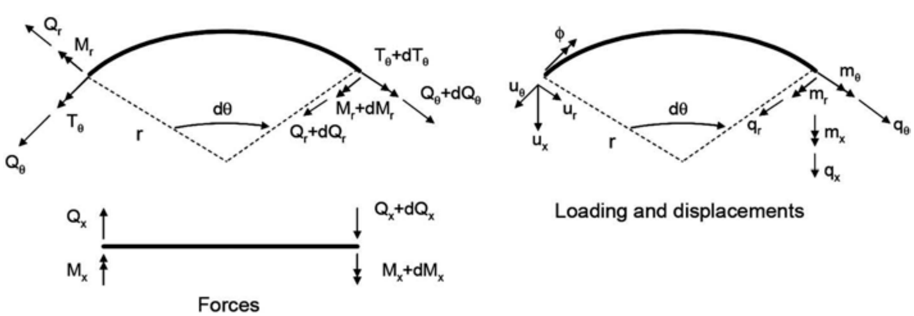

Figure 3. Differential curved beam element and sign conventions.

$$\frac{1}{r^2}\left[\frac{d^3 M_x}{d\theta^3}+\frac{dM_x}{d\theta}\right]=\frac{dq_r}{d\theta} \tag{12}$$

Equation 12 can be solved for the loading condition (q_r) defined in Equation 8 to arrive bending moment in the ring about a transverse axis:

$$M_x(\theta)=-\frac{a_i}{(m^2-1)}\bar{q}\;L\;r^2\cos m\theta \tag{13}$$

The following force-deformation expression was used to obtain radial displacement (u_r):

$$M_x=\frac{EI_x}{r^2}\left(\frac{d^2u_r}{d\theta^2}+u_r\right) \tag{14}$$

where I_x= bending moment of inertia of the ring about transverse axis.
The radial displacement of the ring can be found using Equation 13 as follows:

$$u_r(\theta)=\frac{a_i}{(m^2-1)^2}\frac{\bar{q}\;L\;r^4}{EI_x}\cos m\theta \tag{15}$$

3.2 *Shell stiffness*

The radial shear loading ($q_r(\theta)$) applied to the edge of the shell shown in Figure 4 is carried by what Calladine (1983) terms an edge-string. This non-symmetric load can be transformed into a tangential shear loading ($N_{x\theta}$) at the top edge of the shell as shown in Figure 4.

The membrane theory of shells (Flügge 1973, Calladine 1983, Rotter 1987, Ventsel & Krauthammer 2001) was adopted to obtain the radial displacements of the shell under circumferential shear loading using the edge-string treatment.

Considering the cylindrical shell element shown in Figure 5, the membrane theory of equilibrium equations (Rotter 1987, Ventsel & Krauthammer 2001) are:

$$\frac{\partial N_x}{\partial x}+\frac{1}{r}\frac{\partial N_{x\theta}}{\partial \theta}+p_x=0 \quad \frac{\partial N_{x\theta}}{\partial x}+\frac{1}{r}\frac{\partial N_\theta}{\partial \theta}+p_\theta=0 \quad N_\theta+rp_n=0 \tag{16}$$

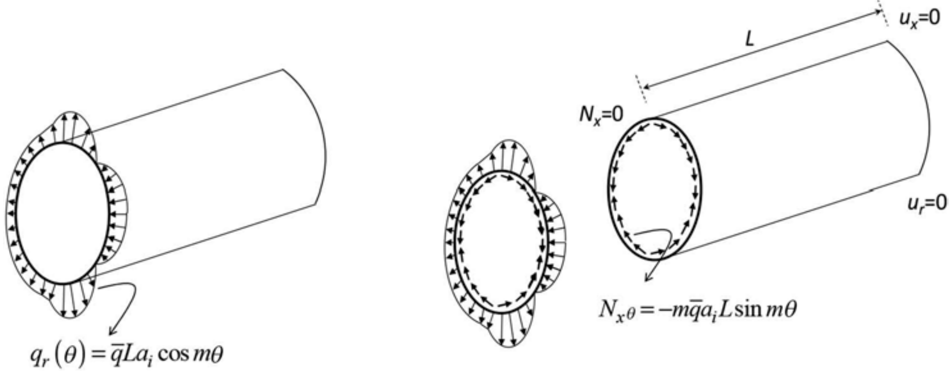

Figure 4. Transferring applied non-uniform radial edge loading to tangential shear loading (Calladine 1983).

where r = middle surface radius; N_x, N_θ, $N_{x\theta}$ = axial, circumferential, and shear membrane stress resultants, respectively; and p_x, p_θ, p_n = external distributed pressures in the axial, circumferential and radial directions, respectively.

The membrane stress resultants (Ventsel & Krauthammer 2001) were given as:

$$N_\theta = -rp_n \; N_{x\theta} = -\int\left(p_\theta + \frac{1}{r}\frac{\partial N_\theta}{\partial \theta}\right)dx + f_1(\theta) \; N_x = -\int\left(p_x + \frac{1}{r}\frac{\partial N_{x\theta}}{\partial \theta}\right)dx + f_2(\theta) \tag{17}$$

where $f_1(\theta)$, $f_2(\theta)$ = unknown functions of θ to be determined from two boundary conditions.

The displacements were found considering strain-displacement relationships as:

$$Etu_x = \int(N_x - \nu N_\theta)dx + f_3(\theta) \; Etu_\theta = 2(1+\nu)\int N_{x\theta}dx - \frac{Et}{r}\int \frac{\partial u_x}{\partial \theta}dx + f_4(\theta) \tag{18}$$

$$Etu_r = Et\frac{\partial u_\theta}{\partial \theta} - r(N_\theta - \nu N_x) \tag{19}$$

where u_x, u_θ, u_r = displacements in the axial, circumferential, and radial directions, respectively; ν = Poisson's ratio; $f_3(\theta)$, $f_4(\theta)$ = additional functions to satisfy the boundary conditions on the edges x = constant.

The initial algebraic treatment here involves no surface loading on the shell ($p_x = p_n = p_\theta = 0$) but involves only tangential shear loading $N_{x\theta}$ at the top (Figure 4). Considering appropriate boundary conditions which were given in Figure 4, the radial displacement can be found as follows at the top of the shell ($x = L$).

$$u_r(\theta) = m^2 a_i \; \bar{q} \; L^2 \; \frac{\left[3r^2(\nu+2) + L^2m^2\right]}{3Etr^2} \; \cos m\theta \tag{20}$$

3.3 *The shell-ring stiffness ratio*

The ratio of the stiffness of the shell to the ring (χ) was found by combining the above expressions. Inserting Equations 15 and 20 into Equation 7 yields the shell-ring stiffness ratio (χ) which can be expressed as:

$$\chi = \frac{u_{r,ring}}{u_{r,shell}} = \frac{1}{m^2(m^2-1)^2}\frac{3r^6t}{LI_x[3r^2(\nu+2) + m^2L^2]} \tag{21}$$

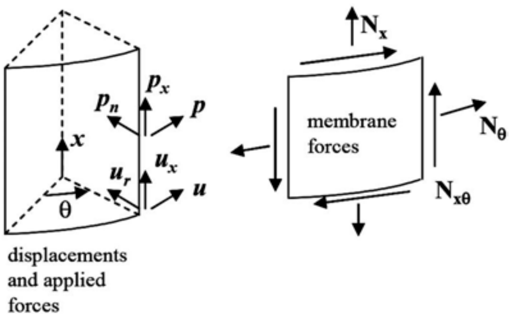

Figure 5. Loading, displacements and stress resultants in an element of the cylindrical shell.

4 BUCKLING BEHAVIOUR OF THE CYLINDRICAL TANK STRUCTURES

4.1 *A brief assessment of each harmonic wind loading*

The effect of each harmonic of loading recommended by EN 1993-4-1 (2007) was investigated separately. A unit pressure was applied at the stagnation point. The commercial finite element program ANSYS v12.1 (2010) was used to perform these numerical analyses. Numeric studies used a constant radius-to-thickness (r/t=1000) ratio and height-to-diameter (L/d_c) ratios of 0.25, 0.5, 0.75, and 1.0 considering different size annular plate ring. The uniform buckling pressure was determined from Donnell theory as follows.

$$q_{cr,D} = 0.92E\left(\frac{t}{r}\right)^2\left(\frac{\sqrt{rt}}{L}\right) \tag{22}$$

From each analysis, the stagnation pressure at buckling ($q_{cr,w}$) was extracted using linear bifurcation analysis. Then, these values were normalised by the buckling uniform pressure obtained from Donnell theory ($q_{cr,w}/q_{cr,D}$). The buckling pressure ratios for each harmonic of loading were plotted in Figure 6 as a function of the shell-ring stiffness ratio.

The comparison shows that the dominant harmonic term is m = 2 for all cases.

4.2 *Linear bifurcation analysis for cylindrical steel tanks with stiffening ring under wind loading*

The linear buckling behaviour of the cylindrical steel storage tanks with top stiffening ring under wind loading was investigated for the same geometries as in the previous part, but r/t changes from 500 to 2500 for all cases. From each analysis, the stagnation pressure at buckling ($q_{cr,w}$) was normalised by uniform pressure obtained from Donnell theory ($q_{cr,w}/q_{cr,D}$). The buckling pressure ratios for wind loading are plotted in Figure 7 as a function of the shell-ring stiffness ratio considering dominant harmonic term of m = 2.

The relationship between buckling pressure ratio and stiffness ratio can be represented by:

$$\frac{q_{cr,w}}{q_{cr,D}} = f(\chi) \tag{23}$$

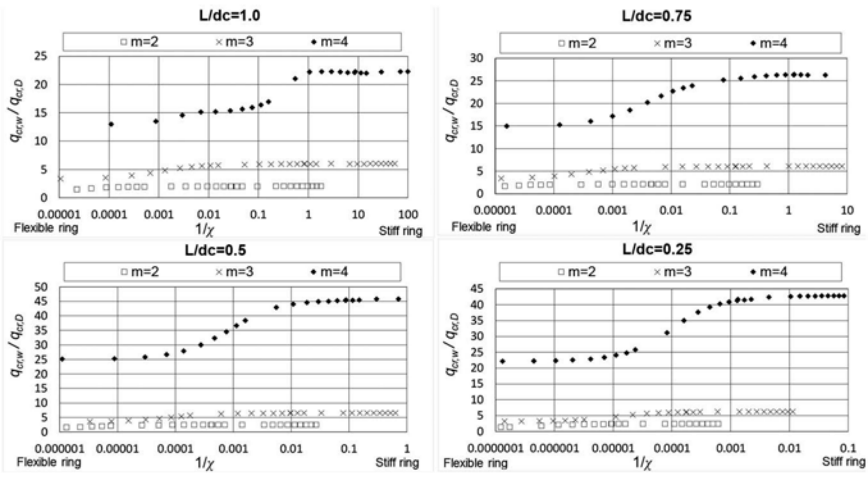

Figure 6. Buckling pressure ratios with different aspect ratios under each harmonic wind loading.

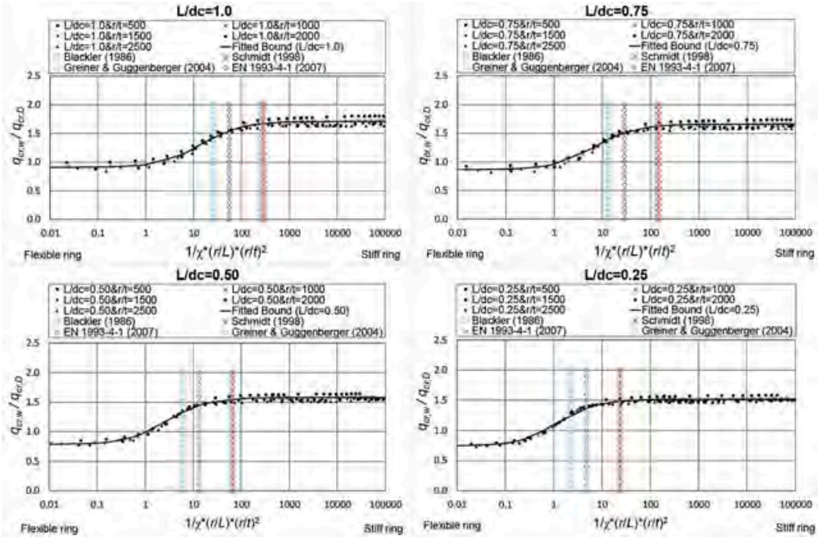

Figure 7. Buckling pressure ratios with proposed equations under wind loading.

Table 1. Coefficients used in the Equation 24.

L/dc	C_1	C_2
1.0	1.71	0.07
0.75	1.66	0.15
0.50	1.58	0.36
0.25	1.52	0.87

where $f(\chi)$ is a function that can be approximated by curve fitting to the data (Figure 7) as

$$f(\chi) = C_1 - \frac{0.8}{1 + \left(\frac{C_2}{\chi}\right)\left(\frac{r}{L}\right)\left(\frac{r}{t}\right)^2} \tag{24}$$

where the C_1 and C_2 = the coefficients given in Table 1.

The proposed expressions considering the shell-ring stiffness ratio are also shown in Figure 7. Obviously, the recommendations given by Schmidt (1998) and Greiner & Guggenberger (2004) are fairly close to each other for design of stiffening rings, but differ from results obtained by Blackler (1986) and EN 1993-4-1 (2007).

5 CONCLUSIONS

This paper has presented a new stiffness requirements of the stiffening ring at the top of the wall of a ground-supported cylindrical tank under wind loading. The buckling resistance of the cylindrical steel storage tanks with top stiffening ring under wind loading that varies along the circumference is investigated using finite element analyses. The changes in the buckling capacity are studied in light of the proposed stiffness ratio that represents the ratio of stiffnesses of the shell and the top ring for a particular harmonic of wind loading. The results revealed that the changes in the buckling capacity are closely related to the shell-top ring stiffness ratio. Furthermore, a generalized solution that shows buckling pressure ratio ($q_{cr,w}/q_{cr,D}$) is then developed as a function of the shell-top ring stiffness ratio.

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