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A New Fuzzy Time Series Model Based on Fuzzy C-Regression Model

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Abstract This study proposes a new fuzzy time series model based on Fuzzy C-Regression Model clustering algorithm (FCRMF). There are two major superiorities of FCRMF in comparison with existing fuzzy time series model based on fuzzy clustering. The first one is that FCRMF partitions data set by taking into account the relationship between the classical time series and lagged values, and thus, it gives the more realistic clustering results. The second one is that FCRMF produces different forecasting values for each data point, while the other fuzzy time series methods produce same forecasting values for many data points. In order to validate the forecasting performance of proposed method and compare it to the other fuzzy time series methods based on fuzzy clustering, six simulation studies and two real-time examples are carried out. According to goodness-of-fit measures, it is observed that FCRMF provides the best forecasting results, especially in cases when time series are not stationary. When considering that fuzzy time series was proposed especially for cases that time series do not satisfy statistical assumptions such as the stationary, this is very important advantage.

Keywords Time series · Fuzzy time series · Fuzzy clustering · Fuzzy C-Regression Model · Forecasting

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1 Introduction

Time series analysis is widely used in many fields including disciplines such as economy, finance, medicine, astronomy and environment science, and various models have been proposed in order to model the behavior of time series. Statistical time series models such as autoregressive model (AR), moving average model (MA), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) are one particular important groups of these models. However, these models are based on strict statistical assumptions. Some of these assumptions are: Time series has to be stationary, error terms have to follow standard normal distribution, and the number of data points of time series should be at least 50. It is very difficult to satisfy these assumptions for real-time series. Besides, statistical time series models cannot deal with forecasting problems in which time series data have uncertainty data points. Therefore, fuzzy time series methods have been getting more and more attractive in recent years.

The definition of fuzzy time series firstly was introduced by Song and Chissom [1–3]. They proposed a fuzzy time series model that consists of four steps: (i) dividing the universe discourse into subintervals, (ii) defining fuzzy sets and fuzzification of classical time series $(Y(t))$, (iii) determining the fuzzy relations between fuzzy sets and (iv) forecasting and defuzzification. Since the studies of Song and Chissom [1–3], a number of fuzzy time series models were proposed to improve the steps of fuzzy time series model proposed by them and enhance the forecasting performance. Sullivan and Woodall [4] analyzed two fuzzy time series methods, first-order time-invariant and time-variant. They compared forecasting results obtained from these models with a time-invariant Markov model and three classical time series models. Chen [5] proposed the new fuzzy time series model to forecast enrollment data of University of Alabama. The method proposed by Chen [5] uses simplified arithmetic operations in step of determining fuzzy relations when comparing with the method proposed by Song and Chissom [1]. Hwang et al. [6] also aimed to simplify the arithmetic operation process. Huarng [7] proposed heuristics models of fuzzy time series. This model integrates heuristic knowledge relating the problem with the model proposed by Chen [5]. Huarng [8] also showed that the selection of length of interval in step of dividing universe discourse highly affects the forecasting performance of fuzzy time series and proposed two methods based on distribution and average of the time series. Huarng and Yu [9] suggested an approach based on ratios, instead of equal lengths of intervals. Yolcu et al. [10] and Eğrioğlu et al. [11] proposed a new approach in order to determine the lengths of intervals based on single-variable constrained optimization. In order to partition universe discourse by taking into account the distributions of data points and consequently improve forecasting accuracy, some studies [12–15] used fuzzy clustering algorithms in the fuzzification step. Using fuzzy clustering algorithms annihilates the problem of determining interval length. Besides, fuzzy time series models can be divided into two groups according to order of fuzzy time series. Most of these studies are based on first-order fuzzy time series. Some studies used higher-order fuzzy time series models [16–21].

This study proposes a new first-order fuzzy time series model based on Fuzzy C-Regression Model (FCRM) clustering algorithm proposed by Hathaway and Bezdek [22]. There are two important advantages of proposed algorithm as follows:

(i) Existing fuzzy time series models based on fuzzy clustering only consider time series itself in clustering process and that successive time data points are independent. This leads to rule out the stochastic relationship between classical time series $(Y(t))$ and its lagged values $(Y(t-1), Y(t-2), \ldots)$ that can be modeled by the statistical models such as AR, MA and ARMA. In the proposed fuzzy time series method, clustering process is carried out by taking into account the stochastic relationship between successive time data points since the cluster center of FCRM that is used the fuzzification step is defined as autoregressive model. Thus, with the use of FCRM, more realistic forecasting results are obtained since both classical and fuzzy relationship in the time series are considered simultaneously.

(ii) In most of fuzzy time series models, forecasting values as much as number of fuzzy relations are obtained. Thus, same forecasting values are obtained for many time data points since fuzzy relations less than the number of data points are obtained. However, proposed method produces different forecasting values for each data point.

(iii) The forecasting performance of existing fuzzy time series models is highly dependent on the length of interval or the number of clusters. In these methods, as the length of interval or the number of clusters increases, the forecasting performance increases. In the proposed method, the forecasting performance is even considerably good in the small number of clusters.

In order to evaluate the forecasting performance of proposed fuzzy time series method, six simulation studies and two real-time examples are carried out. Experimental results show that proposed model gives the better forecasting results when comparing other fuzzy time series models based on fuzzy clustering. The rest of this paper is organized as follows. In Sect. 2, some important definitions related to fuzzy time series are given. In Sect. 3, fuzzy time series methods based on fuzzy clustering is summarized. In Sect. 4, proposed fuzzy time series method is introduced. In Sect. 5, six simulations studies and two real-time examples are carried out in order to evaluate the performance of the proposed method. Section 6 concludes the paper.

2 Basic Concepts of Fuzzy Time Series

In this section, some definitions are given to understand the concepts of fuzzy time series.

Definition 2.1 Let U be the universe of discourse with $U = \{u_1, u_2, \ldots, u_c\}$. A fuzzy set $A_i (i = 1, 2, .., c)$ of U is defined as follows:

$$A_i = \frac{f_{A_i}(u_1)}{u_1} + \frac{f_{A_i}(u_2)}{u_2} + \cdots + \frac{f_{A_i}(u_c)}{u_c}, \quad (1)$$

where $f_{A_i} : U \to [0, 1]$ is the membership function of the fuzzy set A_i, $f_{A_i}(u_r)$ denotes the membership degree of the element u_r to the fuzzy set A_i, $i, r = 1, 2, \ldots, c$.

Definition 2.2 Let $Y(t) \in R^1, t = 0, 1, 2\ldots$ be the universe of discourse defined by fuzzy set A_i, and if $F(t)$ is a collection of $A_i (i = 1, 2, \ldots, c)$, then $F(t)$ is called a fuzzy time series on $Y(t) (t = 1, 2, \ldots)$.

Definition 2.3 If $F(t)$ is caused by $F(t-1)$, then the first-order model relation of $F(t)$ can be represented as $F(t) = F(t-1) o R(t, t-1)$, where $R(t, t-1)$ is a fuzzy relation between $F(t)$ and $F(t-1)$ and "o" denotes the max–min composition operator.

Definition 2.4 Suppose $F(t) = A_i$ and $F(t-1) = A_j$, then fuzzy logical relationship between $F(t)$ and $F(t-1)$ can be represented as $A_i \rightarrow A_j$.

3 Fuzzy Time Series Method Based on Fuzzy Clustering

Fuzzy time series method firstly proposed by Song and Chissom [1–3] consists of four steps: (i) dividing the universe discourse into subintervals (U), (ii) defining fuzzy subsets of the universe discourse and fuzzification of classical time series $(Y(t))$, (iii) deriving fuzzy relations from the fuzzy time series and iv) forecasting and defuzzification.

The steps of dividing universe discourse and fuzzification play an important role in the forecasting performance of fuzzy time series. In the most of fuzzy time series literature [1–3, 5, 7, 8, 16, 19], the process of dividing universe of discourse U is defined as follows: Starting and ending points of U are determined as follows:

$$U = [D_{\min} - D_1, D_{\max} + D_2] = [D_3, D_4], \tag{2}$$

where $D_{\min}$ and $D_{\max}$ are the minimum and maximum values of classical time series data $Y(t)$, respectively, and D_1 and D_2 are two arbitrary values defined by user. The closed interval $[D_3, D_4]$ must contain all values of $Y(t)$. U is partitioned into equal-width intervals according to predefined interval length, and u_i $(i = 1, 2, \ldots, c)$ subintervals are determined. However, this kind of partitioning may not give good forecasting results in cases where the distribution of $Y(t)$ is not uniform. As mentioned in Introduction, some studies use fuzzy clustering algorithm in order to partition universe discourse by taking into account the distributions of data points. General framework fuzzy time series based on fuzzy clustering [12–15] is presented as follows:

Step 1 Dividing the universe discourse into subintervals by using fuzzy clustering.

Fuzzy clustering algorithms such as Fuzzy C-Means [23] and Gustafson-Kessel [24] are applied to classical time series, and it is partitioned into $2 \leq c < n$ number of fuzzy clusters. As a result of fuzzy clustering, c number of cluster centers $(v_1, v_2, \ldots, v_c)$ and membership degrees of data points $(u_{ti} t = 1, 2 \ldots, n, i = 1, 2, \ldots, c$ where n is the number of data points in time series) to these clusters are obtained.

Step 2 Fuzzification of classical time series.

Cluster centers are sorted ascending, and these sorted clusters are used to determine fuzzy sets. The fuzzy sets are represented by $A_i i = 1, 2, \ldots c$. Each data point is assigned to a fuzzy set according to its maximum membership value.

Step 3 Establishing fuzzy relations between the fuzzy sets.

For the first-order fuzzy time series, one lagged fuzzy sets of the fuzzy set at time $t(t = 1, 2, \ldots, n,)$ are obtained. For example, let A_i be fuzzy set at time t and A_j, A_p, A_s be one lagged fuzzy sets of A_i. Then, the fuzzy relation between $F(t)$ and $F(t-1)$ is denoted as $A_i \rightarrow A_j, A_p, A_s$. As a result of this step, fuzzy relation matrix (R) with size (cxc) is obtained. The elements of this matrix are equal to one or zero. For fuzzy relation $A_i \rightarrow A_j, A_p, A_s$, elements r_{ij}, r_{ip} and r_{is} of matrix R are equal to one, and the others are equal to zero.

Step 4 Forecasting and defuzzification.

Forecasting values are obtained at two steps.

Step 4.1 Fuzzy relation matrix (R_{cxc}) is multiplied by cluster center vector (V_{cx1}), and vector RV with size $cx1$ is obtained.

Step 4.2 c number of forecasting values are obtained by using the following equations:

$$\hat{y}_i = \frac{RV_i}{\sum_{j=1}^{c} r_{ij}} i = 1, 2, \ldots c, \tag{3}$$

where if $F(t) = A_i$ $(t = 1, 2, \ldots, ni = 1, 2, \ldots, c)$, then forecasting value of $F(t)$ is equal to $\hat{y}_i$. It is noted that $c < n$ number of forecasting values are obtained for n number of data points. Thus, fuzzy time series method based on FCM and GK does not produce realistic forecasting results. This study proposes to use Fuzzy C-Regression Model clustering algorithm in the fuzzification step, differently from [12, 14].

4 Fuzzy Time Series Method Based on Fuzzy C-Regression Model

This section presents the proposed fuzzy time series method based on Fuzzy C-Regression Model (FCRMF). Firstly, Fuzzy C-Regression Model clustering algorithm used in the fuzzification step is given and then algorithm of proposed method is described.

4.1 Fuzzy C-Regression Model Clustering Algorithm

Fuzzy C-Regression Model (FCRM) clustering algorithm was proposed by Hathaway and Bezdek22 and can be viewed as an extension of *FCM* [23] to linear cluster centers. In the other words, while FCM finds dot-shaped clusters ($f_i = (v_{i1}, v_{i2})$), FCRM finds the linear function-shaped clusters such as $f_i = \phi_{i0} + \phi_{i1} y_{t-1} + \cdots + \phi_{ip} y_{t-p}$.

For first-order autoregressive model, assume the data to be clustered, $Y = \{(y_1, y_2), (y_2, y_3), \ldots, (y_{t-1}, y_t), \ldots, (y_{n-1}, y_n)\}$, come from c number of fuzzy regression models. In this case, the cluster centers of i. cluster can be expressed as autoregressive model [25]:

$$f_i(y_t; \phi_i) = \phi_{i0} + \phi_{i1} y_{t-1} \quad t = 1, 2, \ldots n, \ i = 1, 2, \ldots, c, \tag{4}$$

where $\phi_i (i = 1, 2, \ldots, c)$ is the parameter vector to be estimated, y_t is the current value of the time series, and y_{t-1} is the one lagged values of time series y_t.

FCRM clustering algorithm is based on obtaining the update equations for membership values (u_{ti}) and parameter (ϕ_i) to be minimized the following objective function:

$$J(Y, U, \phi) = \sum_{t=1}^{n} \sum_{i=1}^{c} u_{ti}^{m} (y_t - f_i(y_t; \phi_i))^2, \tag{5}$$

where n is the number of time data points, c is the number of clusters, $1 < m < \infty$ is the fuzziness index, and u_{ti} is the membership degree of t data point to i cluster. u_{ti} has to satisfy the following conditions:

$$u_{ti} \in [0, 1] \ t = 1, 2, \ldots, n \ i = 1, 2, \ldots, c \tag{6}$$

$$0 < \sum_{t=1}^{n} u_{ti} < n \quad i = 1, 2, \ldots, c \tag{7}$$

$$\sum_{i=1}^{c} u_{ti} = 1 \quad t = 1, 2, \ldots, n. \tag{8}$$

Update equations for parameters are as follows:

$$\phi_i = [X_1 W_i X_1]^{-1} X_1^T W_i Y \quad i = 1, 2, \ldots c \tag{9}$$

X_1 and W_i are given in (10) and (11), respectively.

$$X_1 = \begin{pmatrix} 1 & y_1 \\ 1 & y_2 \\ \vdots & \vdots \\ 1 & y_{t-1} \end{pmatrix}, \tag{10}$$

where $y_1, y_2, \ldots, y_{t-1}$ is the successive elements of the time series under consideration:

$$W_i = \begin{pmatrix} u_{1i} & 0 & \ldots & 0 \\ 0 & u_{2i} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & u_{ni} \end{pmatrix}, \tag{11}$$

where u_{ti} is calculated as follows:

Table 1 Working principle of FCRM

Step 1 Initialization
Determining initial values such as number of clusters c, fuzziness index m, termination criteria ε and initial membership matrix U
Step 2 Estimate parameters $\phi_i (i = 1, 2, .., c)$ by using Eq. (9)
Step 3 Calculate membership degrees u_{ti} by using Eq. (12)
Step 4 If $\lvert U^r - U^{r-1} \rvert < \varepsilon$, then terminate algorithm, otherwise go to Step 2 (where r is iteration number)
Step 5 Calculate the value of cluster center for each data point based on Eq. (4)

$$u_{ti} = \left(\sum_{k=1}^{c} \frac{(y_t - f_i(y_t; \phi_i))^2}{(y_t - f_k(y_t; \phi_k))^2} \right)^{\frac{1}{m-1}} \quad t = 1, 2, \ldots, n, \ i = 1, 2, \ldots, c. \tag{12}$$

The working principle of FCRM is summarized in Table 1.

As can be seen in Table 1, FCRM is carried out through an iterative minimization of the objective function given in (5) with the update of the parameter vectors computed in (9) and membership degree in (12).

4.2 Algorithm of the Proposed Method

In this section, fuzzy time series method based on FCRM clustering algorithm is proposed. The proposed algorithm consists of four steps as is the other fuzzy time series methods.

Step 1 Apply FCRM to classical time series $y(t)$.

FCRM presented in Sect. 4.1 with number of clusters c is applied to classical time series data ($y(t)$). As a result of FCRM, membership degrees u_{ti} ($t = 1, 2, \ldots, n \ i = 1, 2, \ldots, c$) to be utilized in the fuzzification step and parameters $\phi_i (i = 1, 2, \ldots, c)$ and cluster centers $f_i(y_t; \phi_i)$ ($t = 1, 2, \ldots, ni = 1, 2, \ldots, c$) given in Eq. (4) are obtained.

Step 2 Fuzzification.

In this step, classical time series (Y_t) is transformed to fuzzy time series ($F(t)$). For this purpose, firstly, the cluster number related to maximum membership degree is found for each time data point. For example, if k time data point belongs to cluster i with maximum membership degree, fuzzy equivalent of k time data point is $F(k) = A_i$, where A_i is the i fuzzy set.

Step 3 Defining fuzzy relations.

For the first-order model, fuzzy set that corresponds to $F(t-1)$ for each $F(t) (t = 1, 2, \ldots, n)$ is found. If $F(t) = A_i$ and $F(t-1) = [A_j, A_p, A_s]$, fuzzy relation is expressed as $A_i \rightarrow A_j, A_p, A_s$. Fuzzy relation matrix (R_{cxc}) is established by using these fuzzy relations similar to other fuzzy time series methods. To give an example, at the result of fuzzification operation, let us obtain $F(t) = (A_1, A_1, A_3, A_1, A_2, A_2, A_3)$ for the time series $y(t)$ consisted of 7 time data points and consider the number of clusters is equal to 3. In the circumstances, fuzzy relation is determined as below:

Table 2 Success percentages of FCMF, GKF and FCRMF for non-stationary time series with length 50

No. of clusters	MAPE-training (%)	RMSE-training (%)	MAPE-test (%)	RMSE-test (%)
c = 5				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 10				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 15				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 20				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 25				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100

Table 3 Success percentages of FCMF, GKF and FCRMF for non-stationary time series with length 100

No. of clusters	MAPE-training (%)	RMSE-training (%)	MAPE-test (%)	RMSE-test (%)
c = 5				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 10				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 15				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 20				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 25				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100

Table 4 Success percentages of FCMF, GKF and FCRMF for non-stationary time series with length 150

No. of clusters	MAPE-training (%)	RMSE-training (%)	MAPE-test (%)	RMSE-test (%)
c = 5				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 10				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 15				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 20				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100
c = 25				
FCMF	0	0	0	0
GKF	0	0	0	0
FCRMF	100	100	100	100

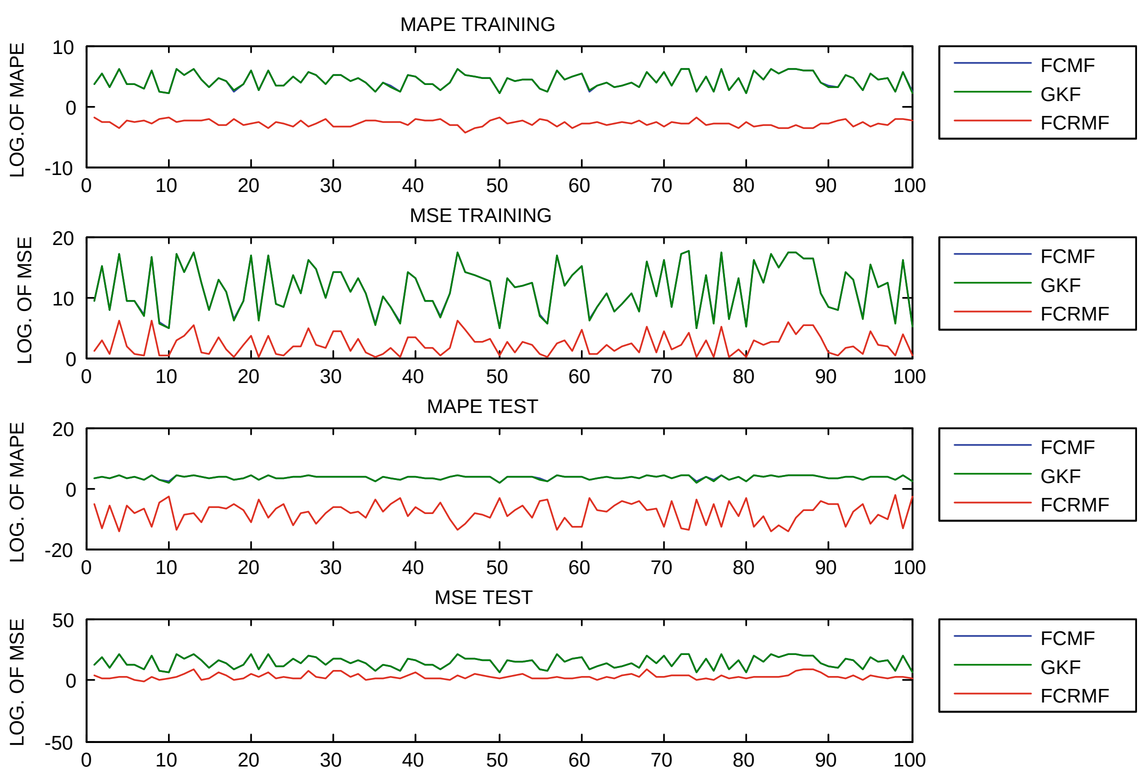

Fig. 1 RMSE and MAPE values for non-stationary time series with length 50 and $c = 25$

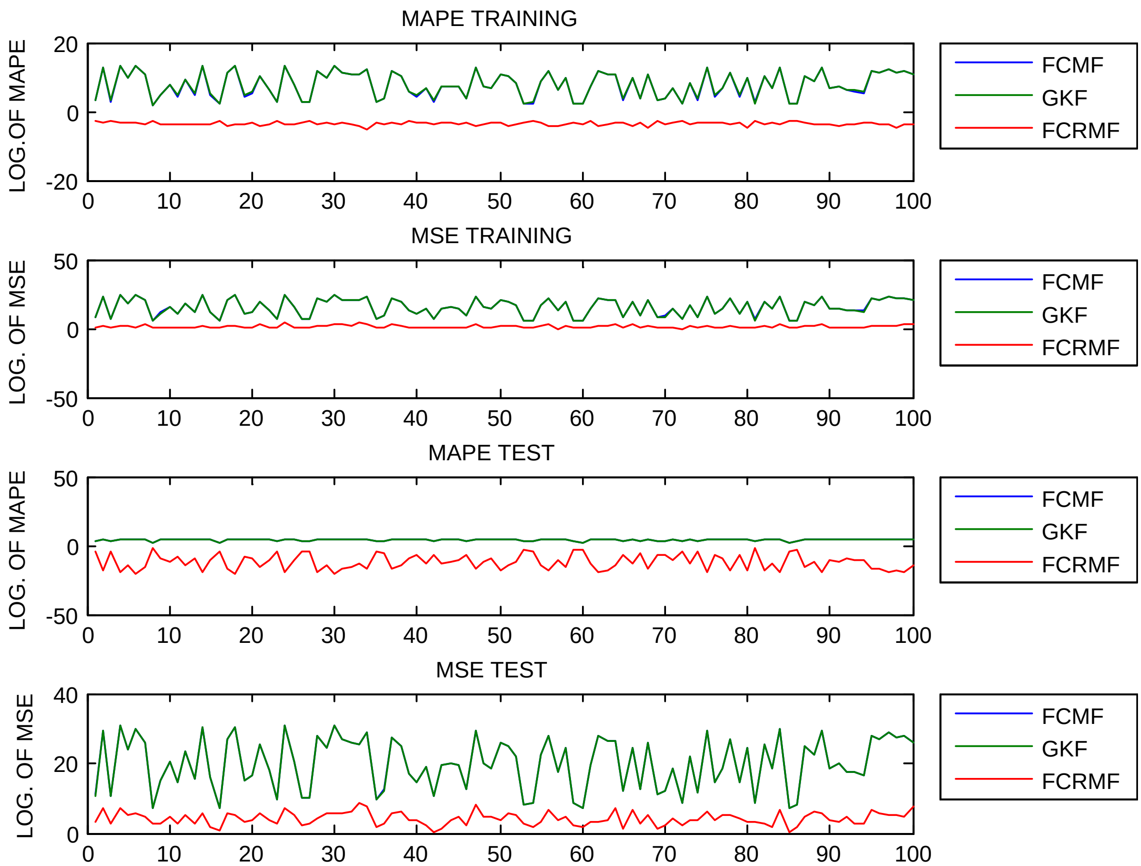

Fig. 2 RMSE and MAPE values for non-stationary time series with length 100 and $c = 25$

$A_1 \rightarrow A_1, A_3 \rightarrow A_1, A_1 \rightarrow A_3, A_2 \rightarrow A_1, A_2 \rightarrow A_2, A_3 \rightarrow A_2$
$A_1 \rightarrow A_1, A_3$
$A_2 \rightarrow A_1, A_2$
$A_3 \rightarrow A_1, A_2$

Fuzzy relation matrix R is obtained as follows:

$$R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}.$$

Step 4 Forecasting and defuzzification.

Step 4.1 Fuzzy relation matrix (R_{cxc}) defined in the previous step is multiplied by cluster center matrix (V_{cxn}), and forecasting matrix (RV) is calculated. V_{cxn} is constructed as follows:

$$V = \begin{pmatrix} f_1(y_1;\phi_1) & f_1(y_2;\phi_1) & \ldots & f_1(y_n;\phi_1) \\ f_2(y_1;\phi_2) & f_2(y_2;\phi_2) & \ldots & f_2(y_n;\phi_2) \\ \vdots & \vdots & \ldots & \vdots \\ f_c(y_1;\phi_c) & f_c(y_2;\phi_c) & \ldots & f_c(y_n;\phi_c) \end{pmatrix}. \quad (13)$$

Step 4.2 If the fuzzy set for k time data point is A_i, then forecasting value for this point is calculated as follows:

$$\hat{y}_k = RV_{ik} / \sum_{j=1}^{c} r_{ij} \quad i = 1, 2, \ldots, c \;\; k = 1, 2, \ldots, n, \quad (14)$$

where r_{ij} is i row and j column element of fuzzy relation matrix (R).

5 Experimental Results

In order to validate the performance of the proposed fuzzy time series model and compare it with the performance of the fuzzy time series methods based on Gustafson-Kessel (GKF) [14] and Fuzzy C-Means (FCMF) [12], six simulation studies and two real-time examples are carried out. In the first real time example, 29 time series consisted of Electricity Consumption Per Capita (ECPC) of Asia countries are used. The other real-time example is implemented on historical enrollment of Alabama University from 1971 to 1992, which has been used in fuzzy time series studies in the literature. In simulation studies, time series with length 50,100 and 150 are generated as follows:

$$Y(t) = \phi_1 Y(t-1) + \varepsilon(t), \quad (15)$$

where $\varepsilon(t)$ follows standard normal distribution and

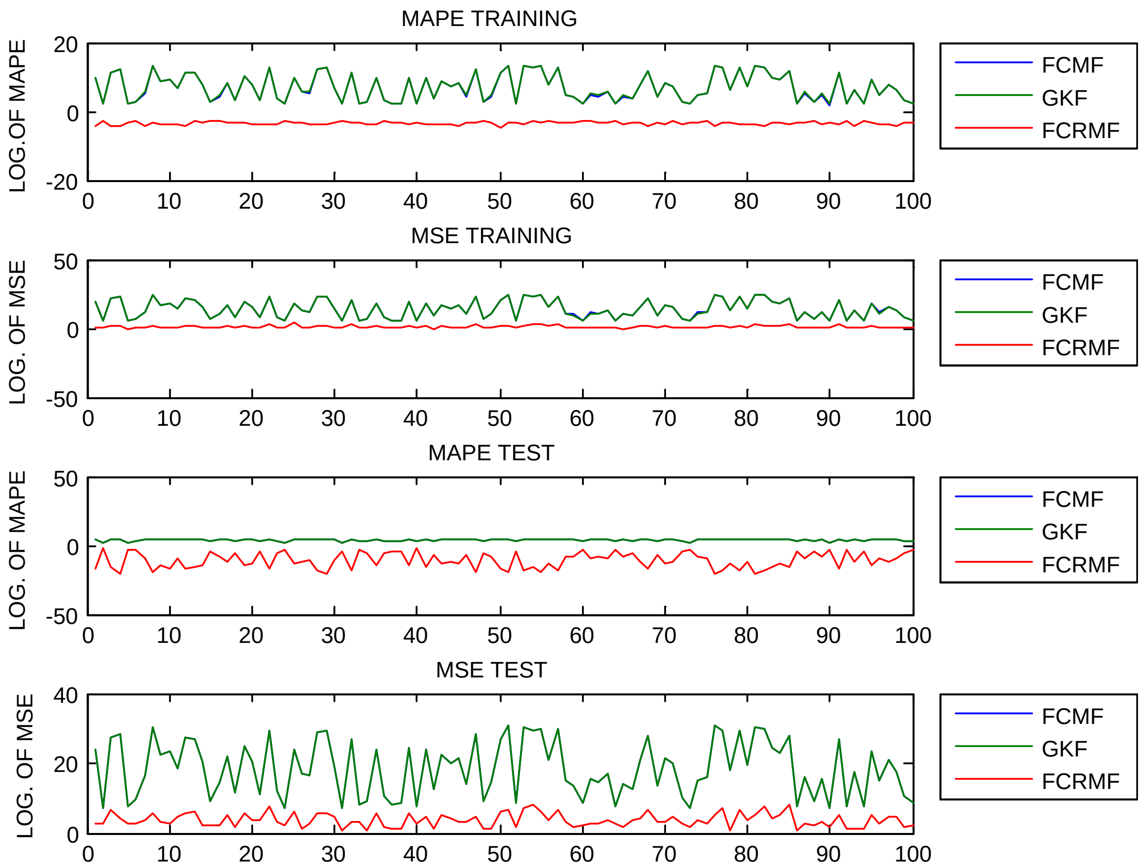

Fig. 3 RMSE and MAPE values for non-stationary time series with length 150 and $c = 25$

Table 5 Success percentages of FCMF, GKF and FCRMF for stationary time series with length 50

No. of clusters	MAPE-training (%)	RMSE-training (%)	MAPE-test (%)	RMSE-test (%)
c = 5				
FCMF	0	0	3	3
GKF	0	0	1	2
FCRMF	100	100	96	95
c = 10				
FCMF	1	1	5	3
GKF	0	0	2	4
FCRMF	99	99	93	93
c = 15				
FCMF	1	2	13	10
GKF	2	1	5	3
FCRMF	97	97	82	87
c = 20				
FCMF	5	5	8	10
GKF	5	7	6	4
FCRMF	90	88	86	86
c = 25				
FCMF	5	5	8	10
GKF	5	7	6	4
FCRMF	90	88	86	86

Table 6 Success percentages of FCMF, GKF and FCRMF for stationary time series with length 100

No. of clusters	MAPE-training (%)	RMSE-training (%)	MAPE-test (%)	RMSE-test (%)
c = 5				
FCMF	0	0	4	3
GKF	0	0	7	3
FCRMF	100	100	89	94
c = 10				
FCMF	0	1	7	6
GKF	0	0	6	5
FCRMF	100	99	87	89
c = 15				
FCMF	0	0	3	4
GKF	1	1	12	8
FCRMF	99	99	85	88
c = 20				
FCMF	0	1	6	4
GKF	1	1	9	6
FCRMF	99	98	85	90
c = 25				
FCMF	3	2	13	9
GKF	3	5	8	3
FCRMF	94	93	79	88

Table 7 Success percentages of FCMF, GKF and FCRMF for stationary time series with length 150

No. of clusters	MAPE-training (%)	RMSE-training (%)	MAPE-test (%)	RMSE-test (%)
c = 5				
FCMF	0	0	2	0
GKF	0	0	3	0
FCRMF	100	100	95	100
c = 10				
FCMF	0	0	6	0
GKF	0	0	2	0
FCRMF	100	100	92	100
c = 15				
FCMF	0	0	4	2
GKF	0	0	8	3
FCRMF	100	100	88	95
c = 20				
FCMF	0	0	13	4
GKF	0	0	9	6
FCRMF	100	100	78	90
c = 25				
FCMF	1	0	7	7
GKF	0	1	15	5
FCRMF	99	99	78	88

$Y(t-1)$ is one lagged value of $Y(t)$ $t = 1, 2, \ldots, n$. For the first three simulation studies, one hundred non-stationary time series $(Y(t))$ are generated with parameters ϕ_1 that vary between 1 and 1.3. For the last three simulation studies, one hundred stationary time series are generated with ϕ_1 that vary between 0.2 and 0.8. FCRMF, GKF and

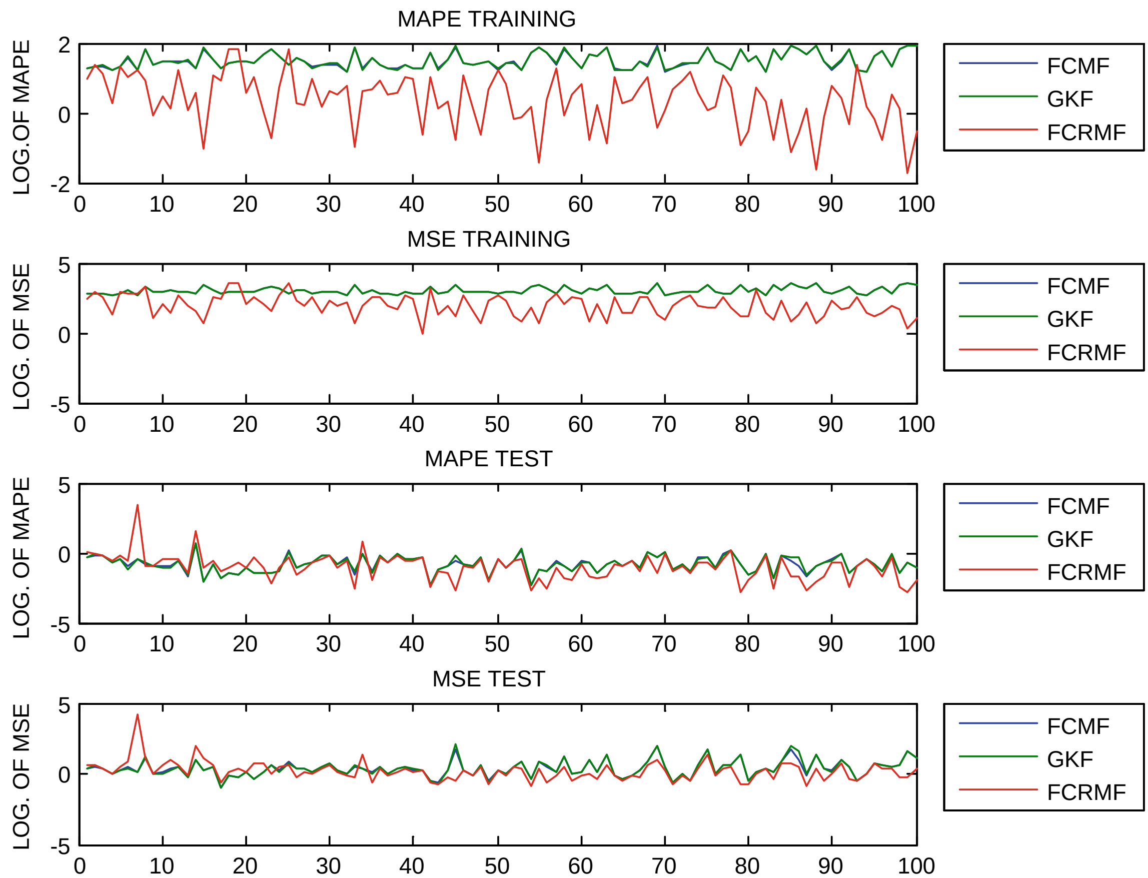

Fig. 4 RMSE and MAPE values for stationary time series with length 50 and $c = 25$

FCMF are applied to these time series with 5, 10, 15, 20 and 25 numbers of clusters, respectively. The reasoning of repeating simulation studies for different lengths of time series is to evaluate the influence of the length of time series on the performance of the proposed method. The goodness-of-fit measures used in the comparisons are mean absolute percentage error (MAPE) and root-mean-square error (RMSE), calculated as follows:

$$\mathrm{MAPE} = \left(\sum_{t=1}^{n} \left| \frac{y_t - \hat{y}_{t-1}}{y_t} \right| \times 100 \right) \Big/ n, \quad (16)$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{n} (y_t - \hat{y}_{t-1})^2}{n}}, \quad (17)$$

where n is the number of data points, y_t is actual value of time series, and $\hat{y}_t$ is the forecasting value. For comparisons, all time series are divided into two mutually exclusive data sets: 95% of time series are designated as the training sets, and the remaining 5% are designated as test sets. Training sets are used to construct fuzzy time series models, and test sets are used to evaluate the performance in long-term forecasting of the models. The results for simulation studies have been given as percentage of success (PS) of each method calculated as follows:

$$\mathrm{PS} = \frac{\text{The number of time series that the method provides best forecasting results}}{\text{Total number of time series (100)}} * 100, \quad (18)$$

where the method having the smallest MSE and MAPE values is determined as the method providing the best forecasting results.

5.1 The Simulation Studies Results for Non-stationary Time Series

Tables 2, 3 and 4 show the PS values defined in Eq. (18) for time series with length 50, 100 and 150, respectively.

As shown in Tables 2, 3 and 4, FCRMF provides one hundred percent success for all cases and the forecasting performance of it does not vary in terms of the length of time series. Besides, as a result of analyses, it has been observed that the performance of GK and FCMF increases as the number of clusters increases. Therefore, in Figs. 1, 2 and 3, MAPE and RMSE values relating to each method have been given for the case that the number of clusters is equal to 25.

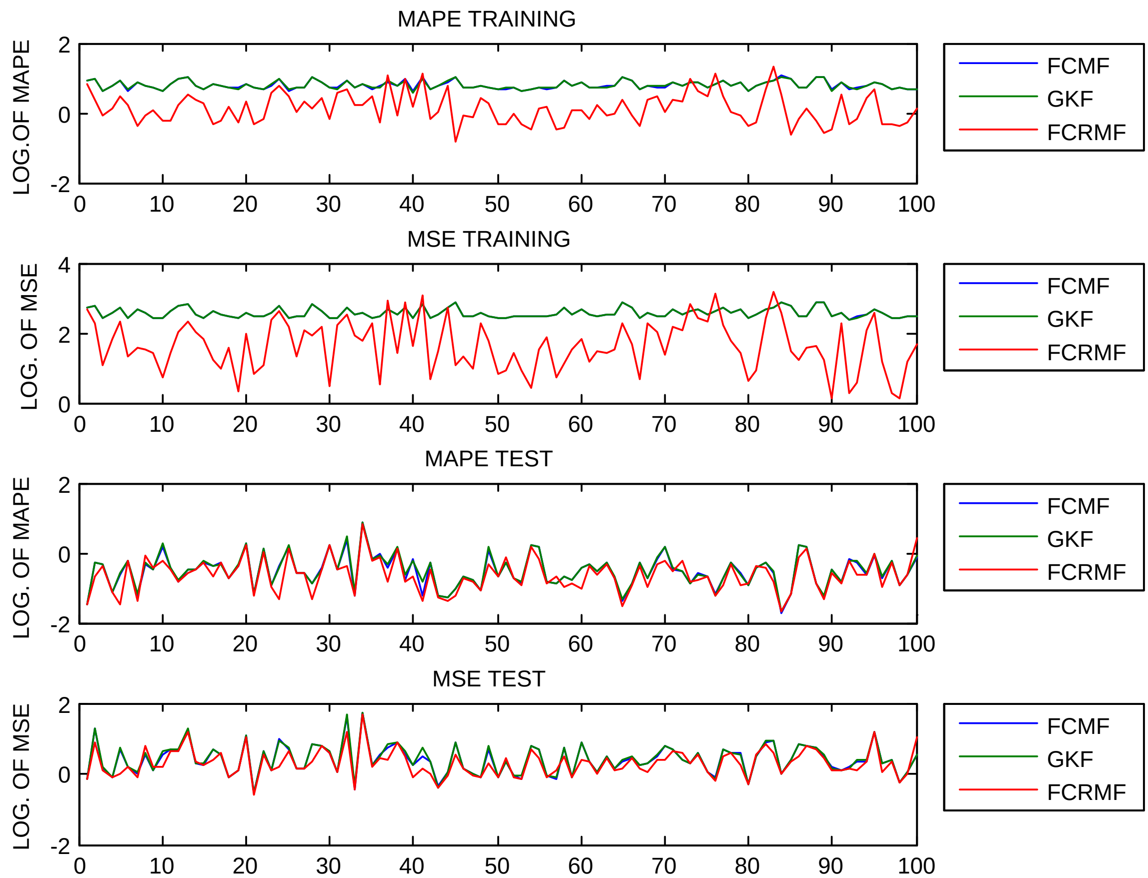

Fig. 5 RMSE and MAPE values for stationary time series with length 100 and $c = 25$

According to Figs. 1, 2 and 3, FCRMF has the smallest MAPE and RMSE values for all cases. Thus, it can be said that the proposed method produces the best forecasting results in even case that GKF and FCMF give their best forecasting results. Besides, the reason that the RMSE and MAPE values of FCMF cannot be seen in Figs. 1, 2 and 3 is that FCMF and GKF give almost the same results.

5.2 The Simulation Studies Results for Stationary Time Series

Tables 5, 6 and 7 present the PS values of each method for stationary time series.

From Tables 5, 6 and 7, it can be seen that the PS values of the proposed method have decreased, while those of GKF and FCMF have increased for especially test sets when comparing with in case of non-stationary time series. The reason of this can be explained as follows. Stationary time series do not show massive increase or decrease with time, and its mean and variance are constant through time. These properties of stationary time series are compatible with GKF and FCMF since they produce same forecasting values for many data points. Figures 4, 5 and 6 denote the RMSE and MAPE values for 100 number of stationary time series.

From Figs. 4, 5 and 6, it can be seen that the performance of FCRMF is better in training sets in comparison with test sets and the performance of the proposed method does not vary according to the length of time series.

5.3 The Results of Real-Time Examples

In this section, GKF, FCMF and FCRMF firstly are applied to ECPC time series of 29 Asia countries and then FCRMF and some existing methods are applied to historical enrollment of Alabama University from 1971 to 1992. Table 8 provides the RMSE and MAPE values for ECPC time series.

In Table 8, the cases that FCMF and GKF methods provide the smallest RMSE and MAPE values are marked as bold. Accordingly, FCRMF gives the best forecasting results for 27 of 29 time series in the case of RMSE-training, all time series in case of MAPE-training, 22 of 29 time series in case of RMSE-test and lastly 23 of 29 time series in case of MAPE-test. Besides, when looking at the RMSE and MAPE values in Table 8, it can be seen that

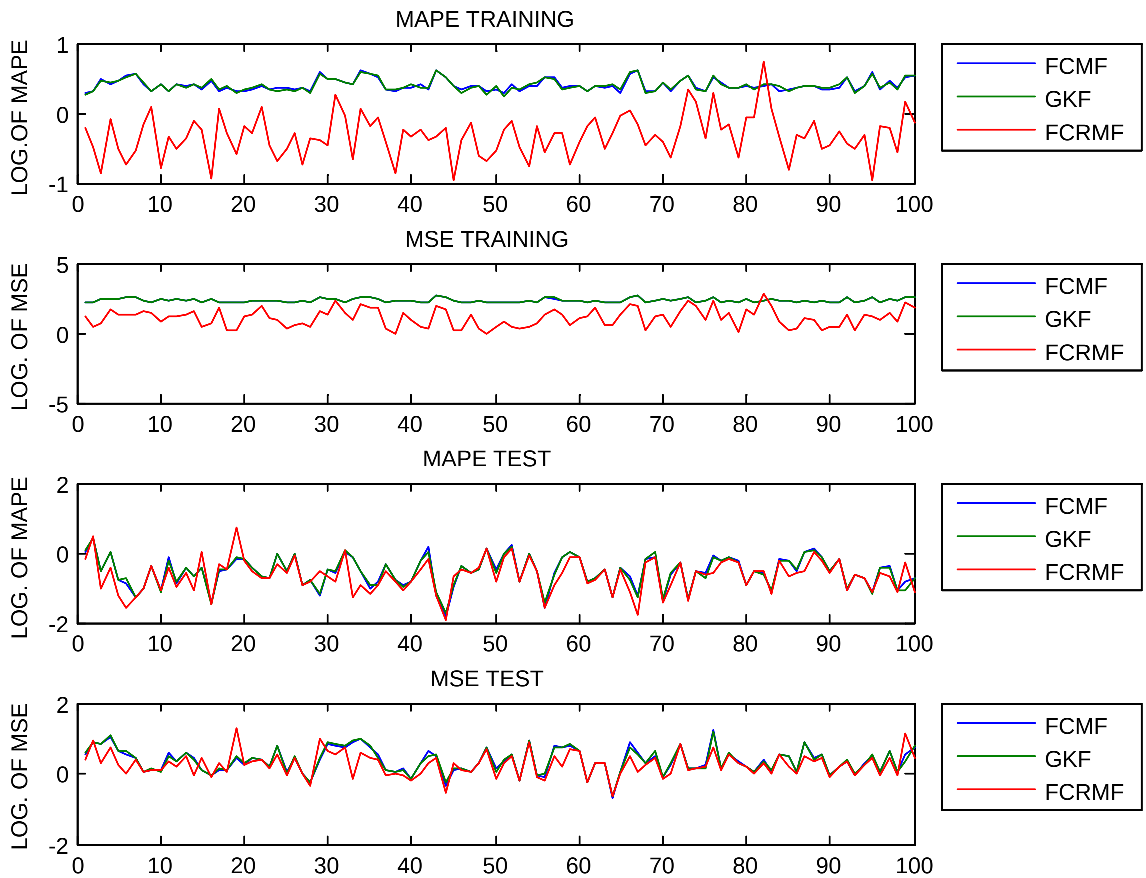

Fig. 6 RMSE and MAPE values for stationary time series with length 150 and $c = 25$

while RMSE and MAPE values of FCRMF are considerably smaller than those of FCMF and GKF in case FCRMF gives the best results, there is no significant difference between RMSE and MAPE values in case FCMF or GKF provides best results. Table 9 shows the comparison results of FCRMF with some existing fuzzy time series methods.

In Table 9, values marked as bold indicate the cases that FCRMF is the best with regard to forecasting performance. Accordingly, it can be said that FCRMF has the best performance and provides a higher forecasting accuracy. Besides, it can be seen that proposed method gives the different forecasting value for each data point, while in other methods same forecasting values are obtained for many data points.

6 Conclusion

In this study, a new fuzzy time series model based on FCRM has been proposed. The major superiorities of proposed model are that it takes into account the relationship between classical time series and its lagged values in clustering process (fuzzification step) and it produces the different forecasting values for each data point. In order to evaluate the performance of the proposed model, six simulations studies are carried out. In the first three simulation studies, one hundred non-stationary time series with length 50, 100 and 150 are generated. Fuzzy time series models based on FCM, GK clustering algorithm and proposed model are applied to these time series with 5, 10, 15, 20 and 25 numbers of clusters, respectively. According to RMSE and MAPE goodness-of-fit measures, it is observed that proposed method gives the best forecasting results for all cases. For the last three simulation studies, one hundred stationary time series are generated and same procedure is repeated for these time series, and it is observed that the performance of proposed model decreases and nevertheless is considerably good when comparing GKF and FCMF. Besides, according to the results of simulation studies, it is concluded that the performance of the proposed model is not affected by the length of time series. Lastly, proposed model and some existing fuzzy time series models are applied to two real-time examples: enrollment data set of Alabama University and electricity power consumption per capita time series of 29 Asia countries. The results have

Table 8 RMSE and MAPE values for ECPC time series of Asia countries

Countries	Method	RMSE-training	MAPE-training	RMSE-test	MAPE-test
Bangladesh	FCMF	18.94	11.85	43.32	104.91
	GKF	18.94	11.85	43.32	104.91
	FCRMF	6.68	2.51	17.36	42.22
Brunei	FCMF	14.54	719.87	14.07	1233.71
	GKF	14.54	719.83	14.07	1233.68
	FCRMF	8.15	291.37	6.28	665.42
China	FCMF	16.16	112.96	49.90	1457.07
	GKF	16.15	112.94	49.90	1457.06
	FCRMF	2.03	12.46	7.29	182.68
Egypt, Arab Rep.	FCMF	15.82	98.92	27.73	457.82
	GKF	15.81	98.91	27.73	457.80
	FCRMF	5.42	27.44	7.43	188.73
India	FCMF	12.55	38.05	29.75	206.27
	GKF	12.54	38.04	29.75	206.26
	FCRMF	3.66	8.04	5.83	46.24
Indonesia	FCMF	29.17	46.07	27.38	188.35
	GKF	29.17	46.06	27.38	188.35
	FCRMF	11.58	9.73	19.92	148.10
Iran	FCMF	14.32	143.76	28.67	753.74
	GKF	14.32	143.74	28.67	753.72
	FCRMF	2.50	21.97	4.94	181.19
Iraq	FCMF	11.33	118.24	27.68	297.15
	GKF	11.33	118.23	27.68	297.16
	FCRMF	**12.39**	97.49	**29.07**	**315.83**
Israel	FCMF	8.81	457.73	7.95	597.66
	GKF	8.81	457.66	7.94	597.59
	FCRMF	3.01	178.10	3.57	313.31
Japan	FCMF	6.75	474.53	4.10	413.37
	GKF	6.75	474.50	4.10	413.37
	FCRMF	2.18	149.62	**4.62**	**493.34**
Jordan	FCMF	12.59	115.02	29.77	674.62
	GKF	12.58	114.98	29.77	674.60
	FCRMF	6.69	40.22	4.10	134.05
Korea	FCMF	23.77	662.79	26.08	2601.19
	GKF	23.77	662.65	26.08	2601.18
	FCRMF	11.47	255.49	16.99	1900.80
Kuwait	FCMF	26.29	1984.88	8.73	1593.60
	GKF	26.29	1984.78	8.73	1593.58
	FCRMF	14.17	1311.47	**14.26**	3024.54
Lebanon	FCMF	16.00	254.07	6.63	255.15
	GKF	16.00	254.06	6.63	255.15
	FCRMF	**16.79**	251.48	**16.95**	**537.48**
Malaysia	FCMF	16.93	251.37	22.26	1009.04
	GKF	16.93	251.32	22.26	1009.04
	FCRMF	11.00	100.10	8.28	452.30
Myanmar	FCMF	10.23	5.34	28.78	43.92
	GKF	15.37	6.70	24.11	40.58
	FCRMF	5.72	3.57	9.35	15.49

Table 8 continued

Countries	Method	RMSE-training	MAPE-training	RMSE-test	MAPE-test
Nepal	FCMF	19.24	5.95	28.47	32.06
	GKF	19.24	5.95	28.47	32.06
	FCRMF	7.88	1.65	3.59	4.76
Oman	FCMF	51.41	406.70	29.23	1765.85
	GKF	51.42	406.64	29.23	1765.84
	FCRMF	32.00	92.28	16.75	1128.65
Pakistan	FCMF	10.96	32.02	18.78	89.20
	GKF	10.96	32.02	18.78	89.20
	FCRMF	6.58	13.75	5.04	31.36
Philippines	FCMF	5.60	26.88	14.13	94.66
	GKF	5.60	26.88	14.13	94.66
	FCRMF	4.72	24.96	13.06	87.04
Qatar	FCMF	16.29	1750.54	7.02	1319.99
	GKF	21.87	2286.50	4.76	846.31
	FCRMF	11.29	893.88	**21.84**	**3846.76**
Saudi Arabia	FCMF	22.14	680.17	20.12	1647.31
	GKF	22.14	680.04	20.12	1647.30
	FCRMF	8.19	124.85	9.08	848.74
Singapore	FCMF	15.10	758.45	12.77	1102.01
	GKF	15.09	758.19	12.76	1101.92
	FCRMF	2.29	119.38	8.50	922.13
Syrian	FCMF	15.85	115.49	22.82	406.25
	GKF	15.86	115.48	22.82	406.23
	FCRMF	7.75	39.83	**26.17**	**578.13**
Thailand	FCMF	18.64	143.65	30.33	686.27
	GKF	21.06	161.40	26.03	603.50
	FCRMF	4.50	35.60	11.78	351.54
Turkey	FCMF	15.26	163.60	29.70	779.61
	GKF	15.26	163.58	29.70	779.60
	FCRMF	2.72	33.74	10.55	283.70
United Arab Emirates	FCMF	18.23	1417.86	8.92	1209.61
	GKF	18.23	1417.69	8.92	1209.62
	FCRMF	4.38	365.92	**26.11**	**3275.03**
Vietnam	FCMF	20.06	31.18	49.66	512.57
	GKF	20.06	31.18	49.66	512.57
	FCRMF	4.22	4.28	17.17	249.89
Yemen	FCMF	13.53	13.56	25.66	59.80
	GKF	13.53	13.56	25.66	59.80
	FCRMF	7.74	6.58	11.55	30.62

Table 9 Comparison of forecasted values of FCRMF with those of some existing methods [12]

Actual	Song and Chissom [3]	Sullivan and Woodall [4]	Chen [5]	Huarng [9]	Cheng et al. [26]	Cheng et al. [12]	FCMF	GKF [14]	Proposed method (FCRMF)
13,055									
13563	14,000	13,500	14,000	14,000	14,320	14,242	13,460	13,477.23	14,416.45
13,867	14,000	14,500	14,000	14,000	14,320	14,242	13,460	13,477.23	14,386.81
14,696	14,000	14,500	14,000	14,000	14,320	14,242	13,460	14,526.08	14,314.89
15,460	15,500	15,321	15,500	15,500	15,541	15,474.3	15,374	15,744.83	15,153.35
15,311	16,000	15,563	15,500	15,500	15,541	15,474.3	15,374	15,744.83	15,543.74
15,603	16,000	15,563	16,000	16,000	15,541	15,474.3	15,374	15,744.83	15,420.57
15,861	16,000	15,500	16,000	16,000	16,196	15,474.3	16,147	15,744.83	15,663.86
16,807	16,000	15,500	16,000	16,000	16,196	16,146.5	16396	16,177.96	**16,530.77**
16,919	16,833	16,684	16,833	17,500	16,196	16,988.3	16,396	16,177.96	**16,907.24**
16,388	16,833	16,684	16,833	16,000	17,507	16,988.3	16,147	16,177.96	**16,227.62**
15,433	16,833	15,500	16,833	16,000	16,196	16,146.5	15,374	15,744.83	15,960.34
15,497	16,000	15,563	16,000	16,000	15,541	15,474.3	15,374	15,744.83	**15,504.86**
15,145	16,000	15,563	16,000	15,500	15,541	15,474.3	15,374	14,526.08	15,561.08
15,613	16,000	15,563	16,000	16,000	15,541	15,474.3	15,374	15,744.83	15,334.96
15,984	16,000	15,563	16,000	16,000	15,541	15,474.3	16,147	15,744.83	15,708.51
16,859	16,000	15,500	16,000	16,000	16,196	16,146.5	16,396	16,177.96	**16,621.3**
18,150	16,833	16,577	16,833	17,500	17,507	16,988.3	16,836	16,781	**17,853.84**
18,970	19,000	19,500	19,000	19,000	18,872	19,144	18,653	18,738.59	18,821.11
19,328	19,000	19,500	19,000	19,000	18,872	19,144	18,653	19,126.77	18,818.88
19,337	19,000	19,500	19,000	19,500	18,872	19,144	18,653	19,126.77	18,865.94
18,876	N/A	N/A	19,000	19,000	18,872	19,144	18,653	18,738.59	18,775.02
RMSE	650.41	621.33	638.36	476.04	511.02	478.45	512.53	467.59	**360.46**
MAPE (%)	3.22	2.66	3.11	2.45	2.66	2.40	2.31	2.20	**1.92**

been demonstrated that FCRMF is considerably good in modeling fuzzy time series.

References

1. Song, Q., Chissom, B.S.: Fuzzy time series and its models. Fuzzy Set Syst. **54**, 269–277 (1993)
2. Song, Q., Chissom, B.S.: Forecasting enrollments with fuzzy time series—part I. Fuzzy Set Syst. **54**, 1–9 (1993)
3. Song, Q., Chissom, B.S.: Forecasting enrollments with fuzzy time series—part II. Fuzzy Set Syst. **62**, 1–8 (1994)
4. Sullivan, J., Woodall, W.H.: A comparison of fuzzy forecasting and Markov model. Fuzzy Set Syst. **64**(3), 279–293 (1994)
5. Chen, S.M.: Forecasting enrollments based on fuzzy time series. Fuzzy Set Syst. **81**(3), 311–319 (1996)
6. Hwang, J.R., Chen, S.M., Lee, C.H.: Handling forecasting problems using fuzzy time series. Fuzzy Set Syst. **100**, 217–228 (1998)
7. Huarng, K.: Heuristics models of fuzzy time series for forecasting. Fuzzy Set Syst. **123**(3), 369–386 (2001)
8. Huarng, K.: Effective lengths of intervals to improve forecasting in fuzzy time series. Fuzzy Set Syst. **123**(3), 387–394 (2001)
9. Huarng, K., Yu, H.K.T.: Ratio-based lengths of intervals to improve fuzzy time series forecasting. IEEE Trans. Syst. Man Cybern. Syst. **36**(2), 328–340 (2006)
10. Yolcu, U., Eğrioğlu, E., Uslu, V.R., Basaran, M.A., Aladağ, C.H.: A new approach for determining the length of intervals for fuzzy time series. Appl. Soft Comput. **9**(2), 647–651 (2009)
11. Eğrioğlu, E., Aladağ, C.H., Basaran, M.A., Yolcu, U., Uslu, V.R.: A new approach based on the optimization of the length of intervals in fuzzy time series. J. Intell. Fuzzy Syst. **22**(1), 15–19 (2011)
12. Cheng, C.H., Cheng, G.W., Wang, J.W.: Multi-attribute fuzzy time series method based on fuzzy clustering. Expert Syst. Appl. **34**, 1235–1242 (2008)
13. Li, S.T., Cheng, Y.C., Lin, S.Y.: A FCM-based deterministic forecasting model for fuzzy time series. Comput. Math Appl. **56**, 3052–3063 (2008)
14. Eğrioğlu, E., Aladağ, C.H., Yolcu, U., Uslu, V.R., Erilli, N.A.: Fuzzy time series forecasting method based on Gustafson–Kessel fuzzy clustering. Expert Syst. Appl. **38**, 10355–10357 (2011)
15. Eğrioğlu, E., Aladağ, C.H., Yolcu, U.: Fuzzy time series forecasting with a novel hybrid approach fuzzy c-means and neural networks. Expert Syst. Appl. **40**, 854–857 (2013)
16. Chen, S.M.: Forecasting enrollments based on high-order fuzzy time series. Cybern. Syst. **33**(1), 1–16 (2002)

17. Chen, S.M., Chung, N.Y.: Forecasting enrollments based on high-order fuzzy time series and genetic algorithms. Int. J. Int. Syst. **21**(5), 485–501 (2006)
18. Jilani, T.A., Burney, S.M.A.: M-factor high order fuzzy time series forecasting for road accident data. Adv. Soft Comput. **34**(1), 328–336 (2007)
19. Lee, L.W., Wang, L.H., Chen, S.W.: Temperature prediction and TAIFEX forecasting based on high-order fuzzy logical relationships and genetic simulated annealing techniques. Expert Syst. Appl. **34**(1), 485–501 (2008)
20. Eğrioğlu, E., Aladağ, C.H., Yolcu, U., Uslu, V.R., Basaran, M.A.: A new approach based on artificial neural network for high order multivariate fuzzy time series. Expert Syst. Appl. **36**(7), 10589–10594 (2009)
21. Eğrioğlu, E., Aladağ, C.H., Yolcu, U., Uslu, V.R., Basaran, M.A.: Finding an optimal interval length in high order fuzzy time series. Expert Syst. Appl. **37**(7), 5052–5055 (2010)
22. Hathaway, R.J., Bezdek, J.C.: Switching regression models and fuzzy clustering. IEEE Trans. Fuzzy Syst. **1**(3), 195–204 (1993)
23. Bezdek, J.C.: Pattern Recognition with Fuzzy Objective Function Algorithms. Plenum, New York (1981)
24. Gustafson, D.E., Kessel, W.C.: Fuzzy clustering with a fuzzy covariance matrix. In: Proc. IEEE Conf. Decision Contr., pp. 761–766 (1979)
25. Runkler, T.A., Seeding, H.G.: Fuzzy c-auto regression models. In: IEEE World Congress on Computational Intelligence, pp. 1818–1825 (2008)
26. Cheng, C.H., Chang, J.R., Yeh, C.A.: Entropy-based and trapezoid fuzzification fuzzy time series approaches for forecasting IT project cost. Technol. Forecast. Soc. **73**, 524–542 (2006)

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